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Unified Representations of Generalized Voigt Function via polynomials and numbers

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Abstract

In this work, we introduce a new generalized form of the Voigt function and investigate its analytical structure through both series and integral representation. These representations are developed in connection with generalized Humbert polynomials, generalized (p,q)-Fibonacci polynomials and several related polynomial families. By establishing these links, we drive a unified framework that brings together a variety of existing results scattered across the literature. The proposed approach not only provides deeper insight into the structural properties of the generalized Voigt function but also reveals new interconnections among special functions and polynomial system.

1 Introduction and Preliminaries

Problems arising in physics and applied mathematics often require efficient numerical evaluation of the Voigt function in both its classical and modified forms, making it an important object of ongoing research. The Function $\mathcal{K}(\Psi,\Lambda)$ and $\mathcal{L}(\Psi,\Lambda)$ play a central role in a wide range of scientific applications. These include, but are not limited to, astrophysical spectroscopy, radiation emission and absorption in heated atmospheres, parameter estimation in electron probe micro analysis, X-ray diffraction line broadening, plasma dispersion phenomena, neutron reaction studies, and several other areas of physical science. Owing to their broad applicability and computational significance, these functions have attracted considerable attention, leading to the development of numerous analytical representations and generalizations. For further details and related studies, the interested reader may consult the works listed in [14, 16, 18, 19, 20, 21, 22, 24, 25, 26, 27, 32, 33, 35].

Extending the work of Srivastava et al. [17], Klusch [7] proposed a generalized formulation of the Voigt functions.

$$\begin{aligned}\mathcal{V}_{\rho,\nu}(\Psi,\Lambda,\Omega) &= \sqrt{\frac{\Psi}{2}}\int_0^\infty t^\rho e^{-\Lambda t-\Omega t^2} J_\nu(\Psi t)dt, \qquad (Re(\Omega)>0, Re(\rho+\nu)>-1) \qquad (1)\\ &= \frac{\Omega^{-\alpha}\Psi^{\nu+\frac{1}{2}}}{2^{\nu+\frac{1}{2}}\Gamma(\nu+1)}\left\{\Gamma(\alpha)\psi_2\left[a;\nu+1,\frac{1}{2};-\frac{\Psi^2}{4\Omega},-\frac{\Lambda^2}{4\Omega}\right]\right\}\\ &\quad -\frac{\Lambda}{\sqrt{\Omega}}\Gamma\left(\alpha+\frac{1}{2}\right)\left\{\psi_2\left[a+\frac{1}{2};\nu+1,\frac{3}{2};-\frac{\Psi^2}{4\Omega},-\frac{\Lambda^2}{4\Omega}\right]\right\}.\end{aligned}$$

Here ψ_2 represents one of Humbert's confluent hypergeometric function of two variables, as described in [15, (p.59)], and is defined as follows:

$$\psi_2[\alpha;\gamma,\gamma';\Psi,\Lambda] = \sum_{\imath,\jmath=0}^{\infty}\frac{(\alpha)_{\imath+\jmath}}{(\gamma)_\imath(\gamma')_\jmath}\frac{\Psi^\imath\Lambda^\jmath}{\imath!\jmath!}, \qquad (\max|\Psi|,|\Lambda|<\infty).$$

Moreover, the classical Bessel function $J_\nu(\Psi)$ [15] is defined as

$$J_\nu(\Psi) = \sum_{\imath=0}^{\infty}\frac{(-1)^\imath\left(\frac{\Psi}{2}\right)^{\nu+2\imath}}{\imath!\Gamma(\nu+\imath+1)}, \qquad (\Psi\in C\backslash(-\infty,0)),$$

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such that,

$$\mathcal{K}(\Psi,\Lambda)=\mathcal{V}_{\frac{1}{2},-\frac{1}{2}}\left(\Psi,\Lambda,\frac{1}{4}\right) \quad \text{and} \quad \mathcal{L}(\Psi,\Lambda)=\mathcal{V}_{\frac{1}{2},\frac{1}{2}}\left(\Psi,\Lambda,\frac{1}{4}\right).$$

The Fibonacci numbers $\mathcal{F}_\jmath$ [17, 28, 29, 30, 31] form a well-Known integer sequence defined recursively by

$$\mathcal{F}_\jmath=\mathcal{F}_{\jmath-1}+\mathcal{F}_{\jmath-2}, \qquad \jmath\geq 2,$$

with initial conditions $\mathcal{F}_0=0$ and $\mathcal{F}_1=1$, yielding the sequence $0,1,1,2,3,5....$ A natural generalization of this sequence is given by the k—Fibonacci numbers $\mathcal{F}_{k,\jmath}$, introduced by Falcon and Plaza [28]. These numbers satisfy the recurrence relation

$$\mathcal{F}_{k,\jmath}=k\mathcal{F}_{k,\jmath-1}+\mathcal{F}_{k,\jmath-2}, \qquad \jmath\geq 2,$$

with the same initial conditions $\mathcal{F}_0=0$ and $\mathcal{F}_1=1$. The formulation extends both the classical Fibonacci sequence and the Pell sequence, the latter being obtained as a special case when $k=2$. Various structural and analytical properties of the k—Fibonacci numbers have been explored in detail in [29].

The Fibonacci polynomials, originally investigated by P. F. Byrd, are defined through the recurrence relation

$$\phi_n(\Psi)=2\Psi\phi_{\jmath-1}(\Psi)+\phi_{\jmath-2}(\Psi), \qquad \jmath\geq 2,$$

with $\phi_0(\Psi)=0$, and $\phi_1(\Psi)=1$. These polynomials extends the Fibonacci sequence into the polynomial framework. In a related development, the Lucas polynomials $\mathcal{L}_\jmath(\Psi)$, first studied by Bicknell in 1970, are defined as follows

$$\mathcal{L}_\jmath(\Psi)=\mathcal{L}_{\jmath-1}(\Psi)+\mathcal{L}_{\jmath-2}(\Psi), \qquad \jmath\geq 2,$$

here $\mathcal{L}_0(\Psi)=2$, and $\mathcal{L}_1(\Psi)=\Psi$.

The $h(\Psi)$—Fibonacci polynomials introduced by Nalli et al. [4], are defined through the generating function

$$\frac{t}{1-h(\Psi)t-t^2}=\sum_{\jmath=0}^{\infty}\mathcal{F}_{h,\jmath}(\Psi)t^\jmath, \qquad \left(|t|<\frac{\sqrt{(h(\psi))^2+4}-|h(\psi)|}{2}\right).$$

For real-valued $h(\psi)$. By selecting specific forms of the function $h(\Psi)$, several well-known sequences emerge as special cases. In particular, choosing $h(\Psi)=\Psi$ yields the Catalan-type Fibonacci polynomials, whereas the choice $h(\Psi)=2\Psi$ leads to the Fibonacci polynomials introduced by Byrd. When $h(\Psi)=k$, the resulting sequence reduces to the k—Fibonacci numbers. Moreover, setting $k=1$ and $k=2$ recovers the classical Fibonacci and Pell numbers, respectively.

For $\imath\geq 2$ be a fixed positive integer, and let $p(\Psi)$ and $q(\Psi)$ be polynomials with real coefficients. The generalized (p,q)—Fibonacci polynomials $\mathcal{U}_{\jmath,\imath}(\Psi)$ and generalized (p,q)—Lucas polynomials $\mathcal{V}_{\jmath,\imath}(\Psi)$, introduced by Wang et al. (see [34]), are defined, respectively, as follows:

$$\mathcal{U}_{\jmath,\imath}(\Psi)=p(\Psi)\mathcal{U}_{\jmath-1,\imath}(\Psi)+q(\Psi)\mathcal{U}_{\jmath-\imath,\imath}(\Psi), \qquad \jmath\geq\imath, \tag{2}$$

where the initial values are specified as

$$\mathcal{U}_{0,\imath}(\Psi)=0,\ \mathcal{U}_{1,\imath}(\Psi)=1,\ \mathcal{U}_{2,\imath}(\Psi)=p(\Psi),...,\mathcal{U}_{\imath-1,\imath}(\Psi)=p^{\imath-2}(\Psi).$$

Also

$$\mathcal{V}_{\jmath,\imath}(\Psi)=p(\Psi)\mathcal{V}_{\jmath-1,\imath}(\Psi)+q(\Psi)\mathcal{V}_{\jmath-\imath,\imath}(\Psi), \qquad \jmath\geq\imath,$$

where the initial values are specified as

$$\mathcal{V}_{0,\imath}(\Psi)=2,\ \mathcal{V}_{1,\imath}(\Psi)=p(\Psi),\ \mathcal{V}_{2,\imath}(\Psi)=p^2(\Psi),...,\mathcal{V}_{\imath-1,\imath}(\Psi)=p^{\imath-1}(\Psi).$$

The generating functions corresponding to the sequences $\mathcal{U}_{\jmath,\imath}(\Psi)$ and $\mathcal{V}_{\jmath,\imath}(\Psi)$ are defined as follows:

$$\mathcal{U}_\imath(\Psi,t)=\sum_{\jmath=0}^{\infty}\mathcal{U}_{\jmath,\imath}(\Psi)t^\jmath=\frac{t}{1-p(\Psi)t-q(\Psi)t^2}, \tag{3}$$

and

$$\mathcal{V}_\imath(\Psi,t)=\sum_{\jmath=0}^{\infty}\mathcal{V}_{\jmath,\imath}(\Psi)t^\jmath=\frac{2-p(\Psi)t}{1-p(\Psi)t-q(\Psi)t^2}, \tag{4}$$

respectively. Assuming real valued polynomial coefficients $p(\psi)$ and $q(\psi)>0$, the expansions converges absolutely inside the open disk $|t|<R$, where

$$R=\frac{\sqrt{(p(\psi))^2+4q(\psi)}-|p(\psi)|}{2q(\psi)}.$$

In addition, they introduced the generalized Humbert polynomials $\mathcal{U}_{\jmath,\iota}^{(\sigma)}(\Psi)$ (p. 205 [34]) as convolutions-type extensions of the polynomials $\mathcal{U}_{\jmath,\iota}(\Psi)$, defined by:

$$\sum_{\jmath=0}^{\infty} \mathcal{U}_{\jmath+1,\iota}^{(\sigma)}(\Psi)t^{\jmath} = \frac{1}{(1-p(\Psi)t-q(\Psi)t^{\iota})^{\sigma}}. \tag{5}$$

The 2-variable Kampé de Feriét generalization of the Hermite polynomials has been studied extensively in the literature (see for details, [9, 10, 11, 12]):

$$\mathcal{H}_{\jmath}(\Psi,\Lambda) = \jmath! \sum_{\iota=0}^{[\frac{\jmath}{2}]} \frac{\Lambda^{\iota}\Psi^{\jmath-2\iota}}{\iota!(\jmath-2\iota)!}, \tag{6}$$

are defined through the associated generating function:

$$e^{\Psi t+\Lambda t^2} = \sum_{\jmath=0}^{\infty} \mathcal{H}_{\jmath}(\Psi,\Lambda)\frac{t^{\jmath}}{\jmath!}. \tag{7}$$

The power series expansion converges absolutely for all $t \in \mathbb{C}$.
When $\Lambda = -1$ and the variable Ψ is replaced by 2Ψ, these polynomials reduce to the classical Hermite polynomials $\mathcal{H}_{\jmath}(\Psi)$ (see, [12]).

More recently, Pathan and Khan introduced the k—Fibonacci-Hermite numbers [23], which are defined through an appropriate generating function:

$$\frac{t}{1-kt-t^2}e^{-t^2} = \sum_{\jmath=0}^{\infty} {}_{\mathcal{H}}\mathcal{F}_{k,\jmath}t^{\jmath}, \qquad (|t| < \frac{\sqrt{k^2+4}-|k|}{2}, k \in \mathbb{R})$$

where ${}_{\mathcal{H}}\mathcal{F}_{k,\jmath}$ denote the Fibonacci-Hermite numbers.

We now recall the definition of h(Ψ)-Fibonacci-Hermite polynomials as introduced in [23], through the following generating function:

$$\frac{t}{1-h(\Psi)t-t^2}e^{\Lambda t+\Omega t^2} = \sum_{\jmath=0}^{\infty} {}_{\mathcal{H}}\mathcal{F}_{h,\jmath}(\Psi,\Lambda,\Omega)t^{\jmath}, \qquad (|t| < \frac{\sqrt{(h(\Psi))^2+4}-|h(\Psi)|}{2})$$

where, h(Ψ) are the polynomials with real coefficients and

$${}_{\mathcal{H}}\mathcal{F}_{h,\jmath}(\Psi,\Lambda,\Omega) = \sum_{\iota=0}^{\jmath} \frac{1}{(\jmath-\iota)!}\mathcal{F}_{h,\iota}(\Psi)\mathcal{H}_{\jmath-\iota}(\Lambda,\Omega). \tag{8}$$

The Pell polynomials [3] are commonly introduced through their generating function

$$\sum_{\jmath=0}^{\infty} \mathcal{P}_{\jmath+1}(\Psi)t^{\jmath} = (1-2\Psi t-t^2)^{-1}, \qquad (|t| < \sqrt{\Psi^2+1}-|\Psi|, \Psi \in \mathbb{R}). \tag{9}$$

The first few Pell polynomials are given by

$$\mathcal{P}_1(\Psi) = 1, \;\; \mathcal{P}_2(\Psi) = 2\Psi, \;\; \mathcal{P}_3(\Psi) = 4\Psi^2+1, \;\; \mathcal{P}_4(\Psi) = 8\Psi^3+4\Psi$$

For $\Psi = 1$, we get $\mathcal{P}_{\jmath}(1) = \mathcal{P}_{\jmath}$, the $\jmath^{th}$-Pell number.

Gegenbauer polynomials $G_{\jmath}^{\lambda}(\Psi)$ [8, 13] are classical polynomials orthogonal to the interval $(-1,1)$ with respect to the weight function $\Psi \to (1-\Psi^2)^{\lambda-1/2} \quad (\lambda > -1/2)$ and are defined by the generating function

$$\frac{1}{(1-2\Psi t-t^2)^{\lambda}} = \sum_{\jmath=0}^{+\infty} G_{\jmath}^{\lambda}(\Psi)t^{\jmath}, \qquad (|t| < \sqrt{\Psi^2+1}-|\Psi|, \Psi \in \mathbb{R}). \tag{10}$$

For $\lambda = 1.5$, first few Gegenbauer polynomials are given by

$$G_0^{1.5}(\Psi) = 1, \;\; G_1^{1.5}(\Psi) = 3\Psi, \;\; G_2^{1.5}(\Psi) = 6\Psi^2 - 1.5...$$

Illustrative Examples

To provide a clearer understanding of the polynomial families used throughout this work, we present some numerical examples for specific parameter choices.

(i) Generalized (p,q)-Fibonacci polynomials: From the recurrence relation

$$U_{j,2}(\psi) = p(\psi)U_{j-1,2}(\psi) + q(\psi)U_{j-2,2}(\psi),$$

with initial values $U_{0,2}(\psi)=0$, $U_{1,2}(\psi)=1$, we choose

$$p(\psi)=\psi, \quad q(\psi)=1.$$

Then the first few polynomials are:

$$U_{0,2}(\psi)=0, \quad U_{1,2}(\psi)=1,$$
$$U_{2,2}(\psi)=\psi, \quad U_{3,2}(\psi)=\psi^2+1,$$
$$U_{4,2}(\psi)=\psi^3+2\psi, \quad U_{5,2}(\psi)=\psi^4+3\psi^2+1.$$

(ii) Generalized Humbert polynomials: Using the generating function

$$\frac{1}{(1-p(\psi)t-q(\psi)t^2)^{\sigma}},$$

and choosing $p(\psi)=\psi$, $q(\psi)=1$, $\sigma=1$, we obtain the same sequence as the (p,q)-Fibonacci polynomials:

$$U^{(1)}_{j+1,2}(\psi)=U_{j,2}(\psi).$$

For $\sigma=2$, the first few terms are:

$$U^{(2)}_{1,2}(\psi)=1, \quad U^{(2)}_{2,2}(\psi)=2\psi,$$
$$U^{(2)}_{3,2}(\psi)=3\psi^2+2, \quad U^{(2)}_{4,2}(\psi)=4\psi^3+8\psi.$$

(iii) Pell polynomials: From the generating function

$$(1-2\psi t-t^2)^{-1},$$

the first few Pell polynomials are:

$$P_1(\psi)=1, \quad P_2(\psi)=2\psi,$$
$$P_3(\psi)=4\psi^2+1, \quad P_4(\psi)=8\psi^3+4\psi.$$

For example, at $\psi=1$, we obtain the Pell numbers:

$$1,\ 2,\ 5,\ 12,\ \ldots$$

(iv) Gegenbauer polynomials: For $\lambda=\frac{3}{2}$, the first few polynomials are:

$$G_0^{3/2}(\psi)=1, \quad G_1^{3/2}(\psi)=3\psi,$$
$$G_2^{3/2}(\psi)=6\psi^2-\frac{3}{2}.$$

For instance, at $\psi=1$, we obtain:

$$G_0^{3/2}(1)=1, \quad G_1^{3/2}(1)=3, \quad G_2^{3/2}(1)=\frac{9}{2}.$$

2 Generalized Voigt function $\Theta^{\sigma}_{\rho,\nu,p,q}(\Psi,\Lambda,\Omega)$

Definition 2.1. Let $p(\vartheta)$ and $q(\vartheta)$ be polynomials with real coefficients and $\sigma\geq 0$. We introduce a generalized Voigt function

$$\Theta^{\sigma}_{\rho,\nu,p,q}[\Psi,\Lambda^{*,\eta}]=\left(\frac{\Psi}{2}\right)^{\frac{1}{2}}\int_0^{\infty}\frac{t^{\rho}e^{\left(-\sum\limits_{j=1}^{\eta}\Lambda_j t^j\right)}}{(1-p(\vartheta)t-q(\vartheta)t^{\iota})^{\sigma}}J_{\nu}(\Psi t)dt \tag{11}$$

$$\big(p\in\mathbb{N};\ \rho,\Lambda^{*,\eta}\in\mathbb{R}^{+};\ \Psi\in\mathbb{R}\ and\ \Re(\rho+\nu>-1)\big),$$

where $\Lambda^{*,\eta}$ abbreviates the array representing $\Lambda_1,\ldots,\Lambda_{\eta}$.

For $\sigma=0$ and $p\eta=2$ in (11), the definition reduces to (1) yeilding

$$\Theta^{\sigma}_{\rho,\nu,p,q}(\Psi,\Lambda,\Omega)\mid_{\sigma=0}=\mathcal{V}_{\rho,\nu}(\Psi,\Lambda,\Omega). \tag{12}$$

Definition 2.2. We also intoduce a generalized Voigt function $\Theta^{\sigma}_{\rho,\nu,p,q}[\Psi,\Lambda,\Omega]$ defined by

$$\Theta^{\sigma}_{\rho,\nu,p,q}[\Psi,\Lambda,\Omega]=\left(\frac{\Psi}{2}\right)^{\frac{1}{2}}\int_0^{\infty}\frac{t^{\rho}e^{-\Lambda t-zt^2}}{(1-p(\vartheta)t-q(\vartheta)t^{\iota})^{\sigma}}J_{\nu}(\Psi t)dt \tag{13}$$

$$\big(\rho,\Lambda,\Omega\in\mathbb{R}^{+};\ \Psi\in\mathbb{R};\ \Re(\rho+\nu>-1)\big).$$

Similar as above, for $\sigma = 0$ in (13), the definition immediately reduces to (1) which gives

$$\Theta^{\sigma}_{\rho,\nu,p,q}[\Psi,\Lambda,\Omega]\,|_{\sigma=0}=\mathcal{V}_{\rho,\nu}[\Psi,\Lambda,\Omega]$$

and

$$\Theta^{\sigma}_{\rho,\nu,p,q}[\Psi,\Lambda,0]\,|_{\sigma=0}=\left(\frac{\Psi}{2}\right)L\left(t^{\rho}J_{\nu}(\Psi t)\right),$$

where, $L(f(t))$ is the classical Laplace transform [2] of $f(t)$.

The case when $\Omega=\frac{1}{4}$ in (12) yields

$$\Theta^{\sigma}_{1/2,-1/2,p,q}\left[\Psi,\Lambda,\frac{1}{4}\right]\Big|_{\sigma=0}=\mathcal{K}(\Psi,\Lambda)\text{ and }\Theta^{\sigma}_{1/2,1/2,p,q}\left[\Psi,\Lambda,\frac{1}{4}\right]\Big|_{\sigma=0}=\mathcal{L}(\Psi,\Lambda).$$

2.1 Connection with different families of polynomials

Using (5) and (13), we obtain a relationship between the generalized Voigt function $\Theta^{\sigma}_{\rho,\nu,p,q}[\Psi,\Lambda,\Omega]$ and the generalized Humbert polynomials. We have

$$\Theta^{\sigma}_{\rho,\nu,p,q}=\Theta^{\sigma}_{\rho,\nu,p,q}[\Psi,\Lambda,\Omega]=\left(\frac{\Psi}{2}\right)^{\frac{1}{2}}\sum_{\jmath=0}^{\infty}\mathcal{U}^{(\sigma)}_{\jmath+1,\imath}(\vartheta)\int_{0}^{\infty}t^{\rho+\jmath}e^{-\Lambda t-\Omega t^{2}}J_{\nu}(\Psi t)dt, \tag{14}$$

which, in view of (1), can also be written as

$$\Theta^{\sigma}_{\rho,\nu,p,q}[\Psi,\Lambda,\Omega]=\sum_{\jmath=0}^{\infty}\mathcal{U}^{(\sigma)}_{\jmath+1,\imath}(\vartheta)\mathcal{V}_{\rho+\jmath,\nu}(\Psi,\Lambda,\Omega).$$

Corollary 2.1. *With* $\sigma = 1$ *in* (13) *and using* (1), (3), *we obtain*

$$\Theta^{1}_{\rho,\nu,p,q}=\Theta_{\rho,\nu,p,q}[\Psi,\Lambda,\Omega]=\left(\frac{\Psi}{2}\right)^{\frac{1}{2}}\sum_{\jmath=0}^{\infty}\mathcal{U}_{\jmath,\imath}(\vartheta)\int_{0}^{\infty}t^{\rho+\jmath-1}e^{-\Lambda t-\Omega t^{2}}J_{\nu}(\Psi t)dt,$$

where $\mathcal{U}_{\jmath,\imath}(\vartheta)$ *are the* (p,q)*-Fibonacci polynomials given by* (2). *This generate the expression*

$$\Theta_{\rho,\nu,p,q}[\Psi,\Lambda,\Omega]=\sum_{\jmath=0}^{\infty}\mathcal{U}_{\jmath,\imath}(\vartheta)\mathcal{V}_{\rho+\jmath-1,\nu}[\Psi,\Lambda,\Omega]. \tag{15}$$

The formula given by (13) *for* $\Theta_{\rho,\delta,p,q}$ *may be converted to generate*

$$\Theta^{\sigma}_{\rho,\nu,p,q}[\Psi,\Lambda,\Omega]=\sum_{\jmath=0}^{\infty}\mathcal{U}_{\jmath,\imath}(\vartheta)\Theta^{\sigma-1}_{\rho+\jmath-1,\nu,p,q}[\Psi,\Lambda,\Omega],$$

which reduces to (15) *when* $\sigma = 1$.

By specifying the parameters $p(\vartheta)$, $q(\vartheta)$ and $\imath$, the polynomial $\mathcal{U}^{(\sigma)}_{\jmath,\imath}$ reduce to some known polynomials and consequently some more relationship can be obtained. We chose not to mention all but a few given under.

For $p(\vartheta)=2\vartheta$, $q(\vartheta)=1$, $\imath=2$ in (13) and using (1) and (10), we see an interesting relationship between $\Theta^{\sigma}_{\rho,\nu,p,q}(\Psi,\Lambda,\Omega)$, $\mathcal{V}_{\rho,\nu}(\Psi,\Lambda,\Omega)$ and Gegenbauer polynomials $G^{\sigma}_{\jmath}(\vartheta)$ by writing

$$\Theta^{\sigma}_{\rho,\nu,p,q}[\Psi,\Lambda,\Omega]=\left(\frac{\Psi}{2}\right)^{\frac{1}{2}}\sum_{\jmath=0}^{\infty}G^{\sigma}_{\jmath}(\vartheta)\int_{0}^{\infty}t^{\rho+\jmath}e^{-\Lambda t-\Omega t^{2}}J_{\nu}(\Psi t)dt,$$

which gives the formula

$$\Theta^{\sigma}_{\rho,\nu,p,q}[\Psi,\Lambda,\Omega]=\sum_{\jmath=0}^{\infty}G^{\sigma}_{\jmath}(\vartheta)\mathcal{V}_{\rho+\jmath,\nu}[\Psi,\Lambda,\Omega].$$

Similarly, with $\sigma = 1$ in (13) and in view of (9), we obtain a relation with the Pell polynomials $\mathcal{P}_{\jmath}(\vartheta)$ by writing

$$\Theta^{1}_{\rho,\delta,p,q}[\Psi,\Lambda,\Omega]=\Theta_{\rho,\delta,p,q}[\Psi,\Lambda,\Omega]=\left(\frac{\Psi}{2}\right)^{\frac{1}{2}}\sum_{n\jmath=0}^{\infty}\mathcal{P}_{\jmath+1}(\vartheta)\int_{0}^{\infty}t^{\rho+\jmath}e^{-\Lambda t-\Omega t^{2}}J_{\nu}(\Psi t)dt,$$

$$=\Theta_{\rho,\delta,p,q}[\Psi,\Lambda,\Omega]=\sum_{\jmath=0}^{\infty}\mathcal{P}_{\jmath+1}(\vartheta)\mathcal{V}_{\rho+\jmath,\delta}[\Psi,\Lambda,\Omega].$$

3 Explicit representation for $\Theta^{\sigma}_{\rho,\nu,p,q}(\Psi,\Lambda,\Omega)$

By using the definition of 2-variable Kampé de Feriét generalization of the Hermite polynomials given in (7), we obtain the series representation of (13) as

$$\Theta^{\sigma}_{\rho,\nu,p,q}[\Psi,\Lambda,\Omega]]=\left(\frac{\Psi}{2}\right)^{\frac{1}{2}}\sum_{j=0}^{\infty}\frac{1}{j!}\mathcal{H}_j(u,v)\int_0^{\infty}\frac{t^{\rho+j}e^{-(\Lambda+u)t-(\Omega+v)t^2}}{(1-p(\vartheta)t-q(\vartheta)t^{\iota})^{\sigma}}J_{\nu}(\Psi t)dt,$$

$$=\sum_{j=0}^{\infty}\frac{1}{j!}\mathcal{H}_j(u,v)\Theta^{\sigma}_{\rho+j,\nu,p,q}(\Psi,\Lambda+u,\Omega+v),$$

by applying (6) on the right of integral (13).
Since we have the well known result [5]

$$\lim_{\Psi\to 0}\ \Psi^{-\nu}J_{\nu}(\Psi)=\frac{1}{2^{\nu}\Gamma(\nu+1)},$$

a limiting case of (13) can be written as

$$\lim_{\Psi\to 0}\frac{\Theta^{\sigma}_{\rho,\nu,p,q}(\Psi,\Lambda,\Omega)}{\Psi^{\nu+1/2}}=\left\{\frac{1}{2^{\nu+\frac{1}{2}}\Gamma(\nu+1)}\right\}\int_0^{\infty}\frac{t^{\rho+\nu}\ e^{-\Lambda t-\Omega t^2}}{(1-p(\vartheta)t-q(\vartheta)t^{\iota})^{\sigma}}dt$$

$$=\left\{\frac{1}{2^{\nu+\frac{1}{2}}\ \Gamma(\nu+1)}\right\}\sum_{j=0}^{\infty}\frac{1}{j!}\mathcal{H}_j(u,v)\int_0^{\infty}\frac{t^{\rho+j+\nu}\ e^{-(\Lambda+u)t-(\Omega+v)t^2}}{(1-p(\vartheta)t-q(\vartheta)t^{\iota})^{\sigma}}dt. \tag{16}$$

From the representation (16), a number of new expressions originate. One class of expansion can be obtained with the help of (16) and using the result [1, 146(24)]

$$\int_0^{\infty}t^{\rho}e^{-\Lambda t-\Omega t^2}dt=2^{(\rho+1)/2}\Gamma(\rho+1)e^{\Lambda^2/8\Omega}D_{-\rho-1}\left(\sqrt{\frac{\Lambda}{2\Omega}}\right)$$

$$\{\Re(\rho+1)>0,\Re(\Lambda)>0\},$$

where, $D_{-\rho}(\Psi)$ is parabolic cylinder function [15]. Thus we obtain

$$\lim_{\Psi\to 0}\frac{\Theta^{\sigma}_{\rho,\nu,p,q}(\Psi,\Lambda,\Omega)}{\Psi^{\nu+1/2}}=\frac{\Gamma(\tau)}{2^{\nu+\frac{1}{2}}\Gamma(\nu+1)}\ e^{(\Lambda+u)^2/8(\Omega+v)}\sum_{j,k=0}^{\infty}\frac{2^{\frac{\tau+j+k}{2}}(\tau)_{j+k}}{j!}$$
$$\times\ \mathcal{H}_j(u,v)\,\mathcal{U}^{(\sigma)}_{k+1,\iota}(\vartheta)D_{-\tau-j-k}\left(\sqrt{\frac{\Lambda+u}{2(\Omega+v)}}\right),$$

where $\tau=\rho+\nu+1$.

Note that the case of replacing Λ by $\Lambda-u$, Ω by $\Omega-v$ and ρ by $\rho-\nu$, the above result reduce to the following result [6]

$$\int_0^{\infty}t^{\rho}e^{-(\Lambda-u)t-(\Omega-v)t^2}dt=\Gamma(\rho+1)e^{\frac{\Lambda^2}{8\Omega}}\sum_{j=0}^{\infty}\frac{2^{\frac{\rho+j+1}{2}}(\rho+1)_j}{j!}\mathcal{H}_j(u,v)D_{-\rho-j-1}\left(\sqrt{\frac{\Lambda}{2\Omega}}\right)$$

Setting $\sigma=1$ in (16) and using (3) yields

$$\lim_{\Psi\to 0}\frac{\Theta_{\rho,\nu,p,q}[\Psi,\Lambda,\Omega]}{\Psi^{\nu+1/2}}=\left\{\frac{1}{2^{\nu+\frac{1}{2}}\ \Gamma(\nu+1)}\right\}\sum_{j=0}^{\infty}\frac{1}{n!}\mathcal{H}_j(u,v)\int_0^{\infty}\frac{t^{\rho+j+\nu}\ e^{-(\Lambda+u)t-(\Omega+v)t^2}}{(1-p(\vartheta)t-q(\vartheta)t^{\iota})}dt,$$

$$=\left\{\frac{1}{2^{\nu+\frac{1}{2}}\ \Gamma(\nu+1)}\right\}\sum_{j=0}^{\infty}\frac{1}{j!}\mathcal{H}_j(u,v)\sum_{k=0}^{\infty}\mathcal{U}_{k,\iota}(\vartheta)\int_0^{\infty}t^{\rho+j+k+\nu-1}\ e^{-(\Lambda+u)t-(\Omega+v)t^2}dt,$$

$$\frac{\Gamma(\tau)}{2^{\nu+\frac{1}{2}}\Gamma(\nu+1)}e^{(\Lambda+u)^2/8(\Omega+v)}\sum_{j,k=0}^{\infty}\frac{2^{\frac{\tau+j+k}{2}}(\tau)_{j+k}}{j!}$$
$$\times\mathcal{H}_j(u,v)\,\mathcal{U}_{k,\iota}(\vartheta)D_{-\tau-j-k}\left(\sqrt{\frac{\Lambda+u}{2(\Omega+v)}}\right),$$

where $\tau=\rho+\nu$.

Further we introduce a positive integer r to present a slightly modified form of (13).

Definition 3.1. Let $p(\vartheta)$, $q(\vartheta)$ be polynomials with real coefficients, $\sigma \geq 0$ and r be a positive integer. Then

$$\Theta^{\sigma,r}_{\rho,\nu,p,q} = \Theta^{\sigma,r}_{\rho,\nu,p,q}(\Psi,\Lambda,\Omega) = \left(\frac{\Psi}{2}\right)^{\frac{1}{2}} \int_0^\infty \frac{t^\rho e^{-\Lambda t - \Omega t^r}}{(1 - p(\vartheta)t - q(\vartheta)t^\imath)^\sigma} J_\nu(\Psi t)dt \tag{17}$$

$$\left\{\Psi,\Lambda,\Omega \in \Re^+ \text{ and } \Re(\rho+\nu) > -1\right\}.$$

Clearly, the case $r = 2$ corresponds to (13) and we have
$\Theta^{\sigma,2}_{\rho,\nu,p,q} = \Theta^{\sigma}_{\rho,\nu,p,q}(\Psi,\Lambda,\Omega)$ and $\Theta^{\sigma,2}_{\rho,\nu,p,q}(\Psi,\Lambda,\Omega)|_{\sigma=0} = \mathcal{V}_{\mu,\nu}(\Psi,\Lambda,\Omega)$.

Now we consider the explicit representation for the generalized Voigt function (17). Making use of the series representation (4) in (17) and integrating the resulting series term by term with the help of the result

$$\int_0^\infty t^\rho e^{-pt-\beta t^\lambda} dt = \sum_{r=0}^\infty \frac{(-\beta)^r \Gamma(\rho+1+r\lambda)}{r!\; p^{\rho+1+r\lambda}}$$

$$(\Re(\rho+1) > 0,\ \Re(p) > 0 \textit{ and } \lambda > 0),$$

we obtain

$$\begin{aligned}\Theta^{\sigma,r}_{\mu,\nu,p,q}[\Psi,\Lambda,\Omega] &= \left\{\frac{\Psi^{\nu+\frac{1}{2}}}{2^{\nu+\frac{1}{2}}\;\Lambda^{\rho+\nu}}\right\} \sum_{\jmath=0}^\infty \frac{\mathcal{U}_{\jmath,\imath}(\vartheta)}{\Lambda^n} \\ &\times \left\{\sum_{k,l}^\infty \frac{\Gamma(\rho+\nu+2k+rl+n)}{\Gamma(\nu+1)k!l!}\right\}\left(\frac{-\Omega}{\Lambda^r}\right)\left(\frac{-\Psi^2}{4\Lambda^2}\right)^k.\end{aligned} \tag{18}$$

Formula (18) is an interesting generalization of a representation [22, p.13,(4.3)] in terms of Kampé de Feriét series $\mathcal{F}^{p:q;r}_{l:\imath;\jmath}$ (see [15, p.63])

$$\begin{aligned}\Theta_{\mu,\nu}[\Psi,\Lambda,\Omega] &= \frac{\Psi^{\nu+\frac{1}{2}}\Gamma(\mu+\nu+1)}{2^{\nu+\frac{1}{2}}\;\Lambda^{\mu+\nu+1}\Gamma(\nu+1)} \\ &\times \mathcal{F}^{2:0;0}_{0:1;0}\left[\begin{array}{ll} \frac{\mu+\nu+1}{2}, \frac{\mu+\nu+2}{2} \quad : \underline{\quad\quad};\underline{\quad}\;; & \\ & \left|\frac{-\Psi^2}{\Lambda^2}, \frac{-4\Omega}{\Lambda^2}\right. \\ \underline{\quad\quad\quad} \quad\quad : \nu+1; \underline{\quad}\;; & \end{array}\right],\end{aligned} \tag{19}$$

which is given recently for $\Omega = \frac{1}{4}$, by Pathan and Shawan [19].
The representation (19) is derivable from (18) by setting $r = 2$ and $\sigma = 0$ and then using Legendre's duplication formula (p. 23 [15]).

4 Representation of $\Theta^\sigma_{\rho,\nu,h}(\Psi,\Lambda,\Omega)$

Theorem 4.1. *For $s \geq 1$*

$$\Theta^\sigma_{\mu,\nu,h}[\Psi,\Lambda,\Omega] = \sum_{\jmath=0}^\infty \sum_{s=0}^{\frac{[\jmath]}{\imath}} \binom{\sigma + s\text{-}1}{s} \binom{\sigma + \jmath - (\imath - 1)\, s\text{-}1}{\jmath - \imath\, s} p^{\jmath - \imath s}(\vartheta) q^s(\vartheta) \mathcal{V}_{\rho+\jmath,\nu}[\Psi,\Lambda,\Omega]. \tag{20}$$

Proof. We have

$$G(p(\vartheta),q(\vartheta),t) = \frac{1}{(1 - p(\vartheta)t - q(\vartheta)t^\imath)^\sigma} = \sum_{\jmath=0}^\infty \mathcal{U}^\sigma_{\jmath+1,\imath}(\vartheta)t^\jmath,$$

and the well known result (Eq. 3.2 [34])

$$\mathcal{U}^\sigma_{\jmath+1,\imath}(\vartheta) = \sum_{s=0}^{[\frac{\jmath}{\imath}]} \binom{\sigma+s-1}{s}\binom{\sigma+\jmath-(\imath-1)s-1}{\jmath-\imath s} p_{\jmath-\imath s}(\vartheta)q^s(\vartheta). \tag{21}$$

Now in view of (14), we write

$$\Theta^\sigma_{\rho,\nu,p,q}[\Psi,\Lambda,\Omega] = \left(\frac{\Psi}{2}\right)^{\frac{1}{2}} G(p(\vartheta),q(\vartheta),t) \int_0^\infty t^{\mu+\jmath} e^{-\Lambda t - z t^2} J_\nu(\Psi t)dt,$$

which, on using (1) and (21) gives (20).

Further, by using the explicit representation of the generalized (p,q)-Fibonacci polynomials $\mathcal{U}_{\jmath,\imath}(\vartheta)$ (Eq. 2.6 [34])

$$\mathcal{U}_{\jmath,\imath}(\vartheta) = \sum_{s=0}^{[\frac{\jmath-1}{\imath}]} \binom{\jmath-1-(\imath-1)s}{s} p^{\jmath-1-\imath s}(\vartheta)q^s(\vartheta) \qquad \jmath \geq 0$$

we obtain another representation given by the following Theorem.

Theorem 4.2. *For $s \geq 1$*

$$\Theta_{\rho,\nu,p,q}[\Psi,\Lambda,\Omega]=\sum_{\jmath=0}^{\infty}\sum_{s=0}^{[\frac{\jmath-1}{\imath}]}\binom{\jmath-1-(\imath-1)s-1}{s}p^{\jmath-1-\imath s}(\vartheta)q^{s}(\vartheta)\mathcal{V}_{\rho+\jmath-1,\nu}[\Psi,\Lambda,\Omega]. \tag{22}$$

Proof. Following the proof of theorem 4.1, the required result (22) can easily be obtained. □

5 Another representation for $\Theta^{\sigma}_{\mu,\nu,p,q}(\Psi,\Lambda,\Omega)$

Using the result [5]

$$\begin{aligned}\int_0^{\infty} t^{\alpha}J_{\nu}(\Psi t)dt = {} & \frac{sin\,\nu\pi\Gamma(\alpha+1)/2}{2\,\nu\pi\;\Omega^{(\alpha+1)/2}}\;{}_2F_2\left[\;1,\tfrac{\alpha+1}{2};\;1+\tfrac{\nu}{2},1-\tfrac{\nu}{2};\;\Big|-\frac{\Psi^2}{4\Omega}\right]\\ & -\frac{\Psi sin\,\nu\pi\Gamma(\alpha+2)/2}{2\pi(1-\nu^2)\;\Omega^{(\alpha+2)/2}}\;{}_2F_2\left[\;1,\tfrac{\alpha+2}{2};\;\tfrac{3+\nu}{2},\tfrac{3-\nu}{2};\;\Big|-\frac{\Psi^2}{4\Omega}\right],\end{aligned} \tag{23}$$

we present a new class of series representations of generalized Voigt function $\Theta^{\sigma}_{\mu,\nu,p,q}(\Psi,\Lambda,\Omega)$ associated with the generalized Humbert polynomials $\mathcal{U}^{(\sigma)}_{\jmath+1,\imath}(\vartheta)$ and hypergeometric functions ${}_2F_2$ [15] which takes the form as

$$\begin{aligned}\Theta^{\sigma}_{\mu,\nu,p,q}[\Psi,\Lambda,\Omega]= {} & \left(\frac{\Psi}{2}\right)^{\frac{1}{2}}\sum_{\jmath,k=0}^{\infty}\mathcal{U}^{(\sigma)}_{\jmath+1,\imath}(\vartheta)\frac{(-\Lambda)^k}{k!}\left\{\frac{sin\,\nu\pi\;\Gamma(\rho+\jmath+k+1)/2}{2\,\nu\pi\;\Omega^{(\rho+\jmath+k+1)/2}}\right.\\ & \times{}_2F_2\left[\;1,\tfrac{\rho+\jmath+k+1}{2};\;1+\tfrac{\nu}{2},1-\tfrac{\nu}{2};\;\Big|-\frac{\Psi^2}{4\Omega}\right]\\ & \left.-\frac{\Psi sin\,\nu\pi\Gamma(\rho+\jmath+k+2)/2}{2\pi(1-\nu^2)\;\Omega^{(\rho+\jmath+k+2)/2}}\;{}_2F_2\left[\;1,\tfrac{\rho+\jmath+k+2}{2};\;\tfrac{3+\nu}{2},\tfrac{3-\nu}{2};\;\Big|-\frac{\Psi^2}{4\Omega}\right]\right\}.\end{aligned} \tag{24}$$

Using (5), (23) and expanding the exponential function $e^{\Lambda t}$ in (13), we can easily prove (24).

A consequence of (24), when $\Lambda=0$ is

$$\begin{aligned}\Theta^{\sigma}_{\mu,\nu,p,q}[\Psi,0,\Omega]= {} & \left(\frac{\Psi}{2}\right)^{\frac{1}{2}}\sum_{\jmath=0}^{\infty}\mathcal{U}^{(\sigma)}_{\jmath+1,\imath}(\vartheta)\left\{\frac{sin\,\nu\pi\Gamma(\rho+\jmath+1)/2}{2\,\nu\pi z^{(\rho+\jmath+1)/2}}\right.\\ & \times{}_2F_2\left[\;1,\tfrac{\rho+\jmath+1}{2};\;1+\tfrac{\nu}{2},1-\tfrac{\nu}{2};\;\Big|-\frac{\Psi^2}{4\Omega}\right]\\ & \left.-\frac{\Psi sin\,\nu\pi\Gamma(\rho+\jmath+2)/2}{2\pi(1-\nu^2)\;z^{(\rho+\jmath+2)/2}}\;{}_2F_2\left[\;1,\tfrac{\rho+\jmath+2}{2};\;\tfrac{3+\nu}{2},\tfrac{3-\nu}{2};\;\Big|-\frac{\Psi^2}{4\Omega}\right]\right\}.\end{aligned}$$

For $\sigma=1$ and using (3), (24) associates with generalized (p,q)-Fibonacci polynomials $\mathcal{U}_{\jmath,\imath}(\vartheta)$ as

$$\begin{aligned}\Theta_{\mu,\nu,p,q}[\Psi,\Lambda,\Omega]= {} & \left(\frac{\Psi}{2}\right)^{\frac{1}{2}}\sum_{\jmath,k=0}^{\infty}\mathcal{U}_{\jmath,\imath}(\vartheta)\frac{(-\Lambda)^k}{k!}\left\{\frac{\sin\,\nu\pi\Gamma(\rho+\jmath+k)/2}{2\,\nu\pi\Omega^{(\rho+\jmath+k)/2}}\right.\\ & \times{}_2F_2\left[\;1,\tfrac{\rho+\jmath+k}{2};\;1+\tfrac{\nu}{2},1-\tfrac{\nu}{2};\;\Big|-\frac{\Psi^2}{4\Omega}\right]\\ & \left.-\frac{\Psi\sin\,\nu\pi\Gamma(\rho+\jmath+k+1)/2}{2\pi(1-\nu^2)\;\Omega^{(\rho+\jmath+k+1)/2}}\;{}_2F_2\left[\;1,\tfrac{\rho+\jmath+k+1}{2};\;\tfrac{3+\nu}{2},\tfrac{3-\nu}{2};\;\Big|-\frac{\Psi^2}{4\Omega}\right]\right\}.\end{aligned}$$

6 The multivariable extension of the Voigt function

The multivariable Hermite polynomials $\mathcal{H}^{(\imath)}_{\jmath}(\{\Psi\}^{\imath}_{1})$, as studied in [9], are characterized through the following generating function:

$$\sum_{\jmath=0}^{\infty}\frac{t^{\jmath}}{\jmath!}\mathcal{H}^{(\imath)}_{\jmath}(\{\Psi\}^{\imath}_{1})=e^{\sum\limits_{j=1}^{\infty}\Psi_j t^j} \tag{25}$$

where $\{\Psi\}^{\imath}_{1}=\Psi_1,\Psi_2,...,\Psi_{\imath}$.
We start by recalling the relationship

$$\mathcal{H}^{(1)}_{\jmath}(\Psi)=\mathcal{H}^{(2)}_{\jmath}(2\Psi,-1)=\mathcal{H}_{\jmath}(\Psi)$$

where $\mathcal{H}_{\jmath}(\Psi_1,\Psi_2)$ denote the two-variable Hermite-Kampé de Feriét polynomials, as define in (5).

The three-variable generalized Hermite polynomials $\mathcal{H}_\jmath(\Psi,\Lambda,\Omega)$, introduced via the generating function [11, p. 511]), are given by:

$$\sum_{\jmath=0}^{\infty}\frac{t^\jmath}{\jmath!}\mathcal{H}_\jmath(\Psi,\Lambda,\Omega)=e^{2\psi t-\Lambda t^2+\Omega t^3}, \tag{26}$$

where $\mathcal{H}_\jmath^{(3)}(2\Psi,-\Lambda,\Omega)=\mathcal{H}_\jmath(\Psi,\Lambda,\Omega)$.

Among the various special cases of (6), examined by Dattoli et al. [10], we highlight the following generating function:

$$\sum_{\jmath=0}^{\infty}\frac{t^\jmath}{\jmath!}h_\jmath(\Psi,\Lambda;\xi)=e^{2\Psi t-t^2+2\Lambda\xi t-\xi^2t^2}, \tag{27}$$

which yields the following expansion in terms of ordinary Hermite polynomials

$$\mathcal{H}_\jmath^{(2)}(2\Psi+2\Lambda\xi,-\xi^2-1)=h_\jmath(\Psi,\Lambda;\xi)=\sum_{s=0}^{\jmath}\binom{\jmath}{s}\xi^s\mathcal{H}_{\jmath-s}(\Psi)\mathcal{H}_s(\Lambda) \tag{28}$$

which may equivalently be expressed as

$$h_\jmath(\Psi,\Lambda;\xi)=(1-\xi)^{\jmath/2}h_\jmath\left(\frac{\Psi+\Lambda\xi}{\sqrt{a+\xi^2}}\right).$$

Definition 6.1. Let $p(\vartheta)$ and $q(\vartheta)$ be polynomials with real coefficients, let $\sigma\geq 0$ and let ℓ be a positive integer. The multivariable extension of the Voigt function is then define as:

$$\Upsilon_{\rho,\nu,p,q}^{\sigma,k}=\Upsilon_{\rho,\nu,p,q}^{\sigma,k}[\Psi,\Lambda,\Omega,\Psi_1,\Psi_2,...,\Psi_k]=\left(\frac{\Psi}{2}\right)^{\frac{1}{2}}\int_0^\infty\frac{t^\rho e^{-\Lambda t-\Omega t^2+\sum\limits_{\ell=1}^{k}\Psi_\ell t^\ell}}{(1-p(\vartheta)t-q(\vartheta)t^\iota)^\sigma}J_\nu(\Psi t)dt, \tag{29}$$

where k is a positive integer, $\Psi,\Lambda,\Omega,\Psi_1,\Psi_2,...,\Psi_k\in\Re^+$ and $\Re(\rho+\nu)>-1$.

A comparison of the definitions in (1), (11), (12) and (17) with (29), yields the following relationships:

$$\Upsilon_{\rho,\nu,p,q}^{\sigma,k}[\Psi,\Lambda,0,0,0,\ldots,-\Omega]=\Theta_{\rho,\nu,p,q}^{\sigma,k}[\Psi,\Lambda,\Omega],$$

$$\Upsilon_{\rho,\nu,p,q}^{\sigma,k}[\Psi,\Lambda,\Omega,0,0,\ldots,0]=\Theta_{\rho,\nu,p,q}^{\sigma,k}[\Psi,\Lambda,\Omega],$$

$$\Upsilon_{\rho,\nu,p,q}^{\sigma}[\Psi,\Lambda,\Omega,\Psi_1]=\Theta_{\rho,\nu,p,q}[\Psi,\Lambda-\Psi_1,\Omega],$$

and

$$\Upsilon_{\rho,\nu}[\Psi,\Lambda,\Omega,\Psi_1,\Psi_2]=\Theta_{\rho,\nu}[\Psi,\Lambda-\Psi_1,\Omega-\Psi_2].$$

Moreover, we obtain

$$\lim_{\Psi\to 0}\Upsilon_{\rho,\nu,p,q}^{\sigma,k}[\Psi,\Lambda,\Omega,\Psi_1,\Psi_2,...,\Psi_k]=\frac{1}{\Gamma(\nu+1)}\int_o^\infty\frac{e^{-\Lambda t-\Omega t^2+\sum\limits_{\ell=1}^{k}\Psi_\ell t^\ell}}{(1-p(\vartheta)t-q(\vartheta)t^\iota)^\sigma}\ t^{\rho+\nu}dt,$$

which for $\alpha=0$ reduces to the result [22, p.10(1.9)]

$$\lim_{\Psi\to 0}\Upsilon_{\mu,\nu}[\Psi,\Lambda,\Omega,\Psi_1,\Psi_2,...,\Psi_k]=\frac{1}{\Gamma(\nu+1)}\int_0^\infty t^{\mu+\nu}e^{-\Lambda t-\Omega t^2+\sum\limits_{\ell=1}^{k}\Psi_\ell t^\ell}dt.$$

Using (25), the series expansion of (29) can be obtain. Thus,

$$\Upsilon_{\rho,\nu}^{\sigma,k}[\Psi,\Lambda,\Omega,\Psi_1,\Psi_2,...,\Psi_k]=\left(\frac{\Psi}{2}\right)^{\frac{1}{2}}\sum_{\jmath=0}^{\infty}\frac{1}{\jmath!}\mathcal{H}_\jmath^{(k)}(\{\Psi\}_1^k)\int_0^\infty\frac{t^{\rho+\jmath}e^{-\Lambda t-\Omega t^2}}{(1-p(\vartheta)t-q(\vartheta)t^\iota)^\sigma}J_\nu(\Psi t)dt \tag{30}$$

$$=\sum_{\jmath=0}^{\infty}\frac{1}{\jmath!}\mathcal{H}_\jmath^{(k)}(\{\Psi\}_1^k)\Theta_{\rho+\jmath,\nu}^{\sigma}(\Psi,\Lambda,\Omega),$$

by using (25) in the integral on the right-hand side of (29).

On putting $\alpha=0$, the above equation yields a known result [22, p.14(5.2)].

$$\Upsilon_{\rho,\nu}[\Psi,\Lambda,\Omega,\Psi_1,\Psi_2,...,\Psi_k]=\sum_{\jmath=0}^{\infty}\frac{1}{\jmath!}\mathcal{H}_\jmath^{(k)}(\{\Psi\}_1^k)\Theta_{\rho+\jmath,\nu}[\Psi,\Lambda,\Omega].$$

Our curiosity increases to observe here that by letting $\Psi \to 0$ in the above result, we arrive at

$$\int_o^\infty t^\mu e^{-\Lambda t-\Omega t^2+\sum_{\ell=1}^{k}\Psi_i t^\ell}\,dt=\sum_{\jmath=0}^{\infty}\frac{2^{\jmath/2}(\mu+1)_\jmath}{\jmath!}\mathcal{H}_\jmath^{(k)}(\{\Psi\}_1^k\,D_{-\mu-\jmath-1}\left(\sqrt{\frac{\Lambda}{2z}}\right),$$

where $D_{-\nu}(\Psi)$ is parabolic cylinder function [15].

By setting $k=2, \Psi_1=2\Psi$ and $\Psi_2=-1$ in (30), we have

$$\Upsilon^\sigma_{\rho,\nu}(\Psi,\Lambda,\Omega,2\Psi,-1)=\Theta^\sigma_{\rho,\nu}(\Psi,\Lambda-2\Psi,\Omega+1)=\sum_{\jmath=0}^{\infty}\frac{1}{\jmath!}\mathcal{H}_\jmath(\Psi)\Theta^\sigma_{\rho+\jmath,\nu}(\Psi,\Lambda,\Omega).$$

On going more general with $\iota=3$ in (30), we get

$$\Upsilon^\sigma_{\rho,\nu}(\Psi,\Lambda,\Omega,\Psi_1,\Psi_2,\Psi_3)=\sum_{\jmath=0}^{\infty}\frac{1}{\jmath!}\mathcal{H}_\jmath^{(3)}(\Psi_1,\Psi_2,\Psi_3)\Theta^\sigma_{\rho,\nu}(\Psi,\Lambda,\Omega),$$

where $\mathcal{H}_\jmath^{(3)(\Psi_1,\Psi_2,\Psi_3)}$ is defined by (26).
For $k=2, \Psi_1=2\Psi+\Lambda y\xi$ and $\Psi_2=-\xi-1$, (30) gives

$$\sum_{\jmath=0}^{\infty}\frac{1}{\jmath!}\mathcal{H}_\jmath(\Psi,\Lambda;\xi)\Theta^\sigma_{\rho,\nu}(\Psi,\Lambda,\Omega)=\Theta^\sigma_{\rho,\nu}(\Psi,\Lambda-2\Psi-2\Lambda\xi,\Omega+\xi^2+1)$$

where $\mathcal{H}_\jmath(\Psi,\Lambda;\xi)$ is defined by (27) (or its equivalent form (28)).

7 Conclusion

In this work, we have examined the relationship that arise in the generalized Voigt functions and various families of the polynomials. In particular, we have demonstrated the generalized Voigt function can be represented in the terms of combinations of generalized Humbert polynomials and Kampé de Feriét functions. As a result, the derived relation and the newly obtained generalizations are of significant interest and may prove useful in a variety of future applications. Table 1 presents several special cases of the generalized Humbert polynomials corresponding to the specific parameter choices. Moreover, the results obtain highlight meaningful connection between Voigt functions involving Bessel functions and different polynomial families, suggesting further potential applications through alternative forms of Voigt functions and the associated polynomial structures.

Table 1: Polynomials occurring as special cases of Generalized Humbert polynomials.

Case	σ	$p(\vartheta)$	$q(\vartheta)$	m	$\sum\mathcal{U}_{\jmath,\iota}\frac{t^\jmath}{\jmath!}$	$\mathcal{U}^\sigma_{\jmath,\iota}(\vartheta)$
(1)	1	$p(\vartheta)$	$q(\vartheta)$	2	$\frac{t}{1-p(\vartheta)t-q(\vartheta)t^2}$	(p,q)-Fibonacci $\mathcal{U}_\jmath(\vartheta)$
(2)	1	ϑ	1	2	$\frac{t}{1-\vartheta t-t^2}$	Fibonacci $F_\jmath(\vartheta)$
(3)	1	1	2ϑ	2	$\frac{t}{1-t-2\vartheta t^2}$	Jacobsthal $J_\jmath(\vartheta)$
(4)	1	2ϑ	-1	2	$\frac{t}{1-2\vartheta t+t^2}$	Chebyshev, 2nd Kind $\mathcal{U}_{\iota-1}(\vartheta)$
(5)	1	ϑ	-2	2	$\frac{t}{1-\vartheta t+2t^2}$	Fermat-Horadam, 1st Kind $\phi_\jmath(\vartheta)$
(6)	1	3ϑ	-2	2	$\frac{t}{1-3\vartheta t+2t^2}$	Fermat $\mathcal{F}_n(\vartheta)$
(7)	1	$\vartheta+2$	-1	2	$\frac{t}{1-(\vartheta+2)t+t^2}$	Morgan-Voyce $B_{\jmath-1}(\vartheta)$
(8)	1	ϑ	$-\alpha$	2	$\frac{t}{1-\vartheta t+\alpha t^2}$	Dickson, 2nd Kind $E_{\jmath-1}(\vartheta,\alpha)$
(9)	1	$\vartheta+1$	ϑ	2	$\frac{t}{1-(\vartheta+1)t-\vartheta t^2}$	Delannoy $\mathcal{D}_{\jmath-1}(\vartheta)$
(10)	1	$p(\vartheta)$	1	2	$\frac{t}{1-p(\vartheta)t-t^2}$	Fibonacci-Nalli-Haukkanen $F_{p,\jmath}(\vartheta)$
(11)	1	1	ϑ	m	$\frac{t}{1-t-\vartheta t^\iota}$	Fibonacci-Ma $G_{\jmath,\iota}(\vartheta)$
(12)	1	1	2ϑ	m	$\frac{t}{1-t-2\vartheta t^\iota}$	Jacobsthal-Djordjević $J_{\jmath,\iota}(\vartheta)$
(13)	1	ϑ	-1	ι	$\frac{t}{1-\vartheta t+t^\iota}$	Chebyshev-Djordjević $V_{\jmath,\iota}(\vartheta)$
(14)	1	$\vartheta+2$	-1	ι	$\frac{t}{1-(\vartheta+2)t+t^\iota}$	Morgan-Voyce-Djordjević $B_{\jmath,\iota}(\vartheta)$
(15)	1	$\Psi+1$	$-\Psi$	2	$\frac{t}{1-(\Psi+1)t+\Psi t^2}$	$(\Psi^\jmath-1)/(\Psi-1)$

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