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Comparison of various types of inverse soft covering upper approximations with an application to decision making

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Abstract

This study presents a comparison of various inverse soft covering upper approximations, focusing specifically on the differences between them. A key finding of our work is the introduction of a new type of inverse soft covering upper approximation which is a new approach in developing inverse soft covering based rough sets. We demonstrate that this new approximation is smaller than the three existing approximations described in the literature. Based on this result, we aim to manage the decision-making processes, especially for the uncertainty problems, in a more ideal way.

1 Introduction

In daily life, certain problems involve vague expressions, which cannot be resolved using classical logic tools. This is because classical logic operates within a framework of definiteness and precision. In classical set theory, an element either belongs to a set or it does not—there is no additional information about the elements of the set. Many researchers, stepping beyond the strict and definite nature of classical mathematics, have sought to develop theories capable of addressing problems involving uncertainty. One of the first mathematical models proposed to effectively address problems of uncertainty was fuzzy set theory [1], introduced by Zadeh in 1965. Unlike classical sets that categorize elements as either fully belonging or not belonging to a set, fuzzy set theory assigns a degree of membership to each element. An element may partially belong to a set to a certain degree, reflecting varying levels of uncertainty or vagueness. This model offers a natural structure for generalizing mathematical concepts to fuzzy logic by allowing gradual memberships.

Rough set theory [2], derived from fuzzy logic approaches, has emerged as a significant tool that enables the efficient use of available data, especially when combined with data mining techniques. This theory, based on the equivalence relations to describe uncertainty, was proposed by Pawlak in 1982 [2]. After than several interesting and meaningful extensions of original rough set theory like, covering based, relation based and neighborhood base rough sets have been derived [3, 4, 5, 6]. By organizing incomplete, insufficient, and uncertain information into a form suitable for analysis, rough set theory facilitates the practical application of processes such as rule reduction and classification. This makes it particularly effective for analyzing large-scale databases in real-world scenarios. Considerable amount of works have been done on fundamental results of rough set.

Despite its rapid development, rough set theory has been insufficient for modeling some uncertain problems because it does not consider parameter values. In 1999, Molodtsov [7] introduced a completely new approximation method, soft set theory, which uses the concept of parameters to model such problems. He defined the soft set as a collection of parameterized alternatives by matching desired parameters with the options to be selected. Since soft set theory is especially valuable for its ability to practically express uncertainty problems and for the ease of operations on the encountered uncertainty, it has attracted the interest of many researchers, leading to numerous studies.

Since these above mentioned theories have numerous applications in various fields, many researchers have developed hybrid frameworks and solved decision making problems using these frameworks [8, 9, 10, 11, 12, 13, 14, 15, 16, 17].

In this study, we focus on the inverse soft covering based rough set theory, which can be seen as a generalized rough set model based on inverse soft sets. In 2016, Cetkin et al. [18] initiated the concept of inverse soft set theory and brought a new perspective to the computation of the decision making problems applying the inverse soft sets instead of the soft sets. They give a new approach of inverse soft sets to find the optimal solution comparatively an easier and faster way than the existed algorithms. A possible fusion of rough sets and inverse soft sets is proposed by Demirtaş et al. [19] in 2020. They initiated an inverse soft covering approximation space. They defined inverse soft covering based rough sets which are a combination of inverse soft sets and rough sets by using inverse soft covering rough approximations. They studied three types of inverse soft covering based rough sets and examined the relationships between them. To make this theory more complete, this paper

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proposes fourth type of inverse soft covering based rough sets. Thus we mainly improve the definition of inverse soft covering upper approximation for inverse soft covering based rough sets to make it more reasonable. Because the lower approximations of the existing ones are the same but the upper approximations are different. We show that our inverse soft covering upper approximation is smaller than with other types of inverse soft covering upper approximations. Moreover we develop an algorithm and apply it to a decision-making problem to demonstrate the applicability of the proposed method.

This paper is organized as follows. The following two sections briefly reviews some backgrounds on inverse soft sets, rough sets and inverse soft covering based rough sets. Also in Section 3, some definitions of inverse soft covering based rough sets are revised. In Section 4, the concept of a new type of inverse soft covering upper approximation is presented. Also some of its interesting properties are investigated. By this approximation, a new kind of inverse soft covering based rough sets is defined. In Section 5, this approximation with the three existing approximations described in the literature are compared. In Section 6, an application of multicriteria group decision making is presented. Using inverse soft covering upper and lower approximations are aided this group decision-making process. Section 7 concludes the paper.

2 Preliminaries

Inverse soft covering based rough sets are described by using inverse soft sets and rough sets. Firstly we recall some definitions and results to make this paper self contained.

Definition 2.1. [2] An information system (or a knowledge representation system) is a pair $\wp = (U,A)$ of non-empty finite sets U and A, where U is a set of objects and A is a set of attributes; each attribute $a \in A$ is a function $a : U \to V_a$, where V_a is the set of values (called domain) of attribute a. Specifically, if $\wp = (U,A)$ is an information system and $B \subseteq A$, then an indiscernibility relation $R = I(B)$ can be defined by

$$(x, y) \in I(B) \iff a(x) = a(y), \quad \forall a \in B,$$

where $x, y \in U$, and $a(x)$ denotes the value of attribute a for object x.

Definition 2.2. [2] Let U be a non-empty finite universe and R be an equivalence relation on U. The pair (U,R) is called a Pawlak approximation space. The equivalence relation R is often called an indiscernibility relation and related to an information system. The relation R partitions U into equivalence classes denoted by $\{[x]_R : x \in U\}$, where each equivalence classes $[x]_R = \{y \in U : (x, y) \in R\}$ consists of elements indistinguishable from x with respect to R. For any $X \subseteq U$, using the indiscernibility relation R, one can define the following two operations

$$\underline{R}(X) = \{x \in U : [x]_R \subseteq X\}, \ \ \overline{R}(X) = \{x \in U : [x]_R \cap X \neq \emptyset\}$$

assigning to every subset $X \subseteq U$ two sets $\overline{R}(X)$ and $\underline{R}(X)$ called the R—lower and the R—upper approximation of X, respectively. Moreover, the sets

$$Pos_R X = \underline{R}(X), \quad Neg_R X = U - \overline{R}(X), \quad Bnd_R X = \overline{R}(X) - \underline{R}(X)$$

are referred to as the R—positive, the R—negative and the R—boundary regions of X, respectively. If the R—boundary region of X is empty, i.e., $\underline{R}(X) = \overline{R}(X)$, then X is definable (or crisp) with respect to R. If $Bnd_R X \neq \emptyset$, i.e., $\overline{R}(X) \neq \underline{R}(X)$, then X is said to be rough (or inexact) with respect to R.

Definition 2.3. [7] Let $U = \{u_1, u_2, ..., u_n\}$ be an initial universe set, $E = \{e_1, e_2, .., e_m\}$ be a set of parameters, $A \subseteq E$ and $P(U)$ be the power set of U. The set of ordered pairs

$$(F,A) = \{(e_j, F(e_j) : e_j \in A, F(e_j) \in P(U)\}$$

is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $e_j \in A$, $F(e_j)$ may be considered as the set of e-approximate elements of the soft set (F,A). It is worth noting that $F(e_j)$ may be arbitrary: some of them may be empty, and some may have nonempty intersection [7]. The absence of any restrictions on the approximate description in soft set theory makes it very convenient and easily applicable in practice.

Definition 2.4. [8] Let $U = \{u_1, u_2, ..., u_n\}$ be an initial universe set, $E = \{e_1, e_2, .., e_m\}$ be a set of parameters, $A \subseteq E$ and $P(U)$ be the power set of U. A soft set F_A on the universe U is defined by the set of ordered pairs

$$F_A = \left\{ \left(e_j, f_A(e_j)\right) : e_j \in E, f_A(e_j) \in P(U) \right\}$$

where $f_A : E \to P(U)$, such that $f_A(e_j) \neq \emptyset$, if $e_j \in A \subseteq E$ and $f_A(e_j) = \emptyset$ if $e_j \notin A$. Here, f_A is called an approximate function of the soft set F_A. The value of $f_A(e_j)$ may be arbitrary.

Definition 2.5. [18] Let $U = \{u_1, u_2, ..., u_n\}$ be an initial universe set, $E = \{e_1, e_2, .., e_m\}$ be a set of parameters and $P(E)$ be the power set of E. The set of ordered pairs

$$(\Lambda, U) = \{(u_j, \Lambda(u_j) : u_j \in U, \Lambda(u_j) \in P(E)\}$$

is called an inverse soft set (for short, ISS) over U, where Λ is a mapping given by $\Lambda : U \to P(E)$.

In other words, an inverse soft set over U is a parameterized family of subsets of the parameter set E. Note that for an ISS Λ on U, the subset $\Lambda(u)$ of E denotes the membership parameters of u to the ISS Λ and $E - \Lambda(u)$ denotes the non-membership parameters of u to the ISS Λ. The family of all ISSs on U is denoted by $ISS(U)$.

Definition 2.6. [1] Let U be an initial universe set. A fuzzy set A in U is a set of ordered pairs:

$$A = \{(x, \mu_A(x)) : x \in U\},$$

where $\mu_A : U \longrightarrow [0, 1] = I$ is a mapping and $\mu_A(x)$ (or $A(x)$) states the grade of belongness of x in A. The family of all fuzzy sets in U is denoted by I^U.

Definition 2.7. [12] Let U be an initial universe set, E be a set of parameters and I^U denotes the set of all fuzzy sets on U and $A \subset E$. A pair (f, A) is called a fuzzy soft set over U, where f is a mapping from A into I^U. That is, for each $a \in A$, $f(a) = f_a : U \to I$, is a fuzzy set on U, where $I = [0, 1]$.

Definition 2.8. [18] Let U be an initial universe set, E be a set of parameters and I^E denotes the set of all fuzzy sets on E. A mapping $F : U \to I^E$ is called an inverse fuzzy soft set (for short, $IFSS$) on U.

Note that, the elements of $IFSS$s are fuzzy sets on the parameter set E, i.e., for each $x \in U$, $F_x \triangleq F(x)$ is a fuzzy set on E. An $IFSS$ can be seen as a multi-parameter fuzzy set. In this manner, F_x denotes the function of membership degree of x to the fuzzy soft set F, for each $x \in U$ and $F_x(e)$ denotes the degree of membership of x to the inverse soft set F with respect to the parameter $e \in E$. The family of all $IFSS$s on U is denoted by $IFSS(U)$.

3 Inverse soft covering approximation space

Demirtaş et al. [19] established an inverse soft covering approximation space in 2020 and investigated the concept of inverse soft covering based rough sets which are defined by using inverse soft covering rough approximations. They investigated three types of inverse soft covering based rough set models. They have the same lower approximation but different upper approximations. To begin with, we present basic concepts about these three types of inverse soft covering based rough sets as follows.

Throughout this section, X and E refers to an initial universe and the set of all parameters for X, respectively. Also, A and B denote the subsets of E, otherwise specified. We indicate an inverse soft set over X with $S = (\Lambda, X)$.

Definition 3.1. [19] Let $S = (\Lambda, X)$ be an inverse soft set over X. If $\cup_{x \in X} \Lambda(x) = E$, then S is said to be a full inverse soft set.

Definition 3.2. [19] A full inverse soft set $S = (\Lambda, X)$ over X is called an inverse soft covering set if $\Lambda(x) \neq \emptyset$, $\forall x \in X$. An inverse soft covering set is denoted by IC_S.

Definition 3.3. [19] Let $S = (\Lambda, X)$ be an inverse soft covering set over X. Then the pair $IP = (E, IC_S)$ is called an inverse soft covering approximation space.

Definition 3.4. [19] Let $IP = (E, IC_S)$ be an inverse soft covering approximation space and $a \in E$. Then the soft minimal description of a is defined as follows:

$$IMd_{IP}(a) = \{\Lambda(x) : x \in X \wedge a \in \Lambda(x) \wedge (\forall u \in X \wedge a \in \Lambda(u) \subseteq \Lambda(x) \Rightarrow \Lambda(x) = \Lambda(u))\}.$$

3.1 First type of inverse soft covering based rough sets

Definition 3.5. [19] Let $S = (\Lambda, X)$ be an inverse soft covering set over X and $IP = (E, IC_S)$ an inverse soft covering approximation space. For a set $A \subseteq E$, the inverse soft covering lower and upper approximations are, respectively, defined as

$$\begin{aligned} \underline{IP}_1(A) &= \bigcup_{x \in X} \{\Lambda(x) : \Lambda(x) \subseteq A\}, \\ \overline{IP}_1(A) &= \bigcup_{x \in X} \{\Lambda(x) : \Lambda(x) \cap A \neq \emptyset\}. \end{aligned}$$

A subset $A \subseteq E$ is called first type of inverse soft covering based definable if $\overline{IP}_1(A) = \underline{IP}_1(A)$; in the opposite case, that is, if $\overline{IP}_1(A) \neq \underline{IP}_1(A)$, A is said to be first type of inverse soft covering based rough set.

3.2 Second type of inverse soft covering based rough sets

Definition 3.6. [19] Let $S=(\Lambda,X)$ be an inverse soft covering set over X and $IP=(E,IC_S)$ an inverse soft covering approximation space. For a set $A\subseteq E$, the inverse soft covering lower and upper approximations are, respectively, defined as

$$\begin{aligned}\underline{IP}_2(A) &= \bigcup_{x\in X}\{\Lambda(x):\Lambda(x)\subseteq A\},\\ \overline{IP}_2(A) &= \bigcup\{IMd_{IP}(a):a\in A\}.\end{aligned}$$

A subset $A\subseteq E$ is called second type of inverse soft covering based definable if $\overline{IP}_2(A)=\underline{IP}_2(A)$; in the opposite case, that is, if $\overline{IP}_2(A)\neq\underline{IP}_2(A)$, A is said to be second type of inverse soft covering based rough set.

3.3 Third type of inverse soft covering based rough sets

Definition 3.7. [19] Let $S=(\Lambda,X)$ be an inverse soft covering set over X and $IP=(E,IC_S)$ an inverse soft covering approximation space. For a set $A\subseteq E$, the inverse soft covering lower and upper approximations are, respectively, defined as

$$\begin{aligned}\underline{IP}_3(A) &= \bigcup_{x\in X}\{\Lambda(x):\Lambda(x)\subseteq A\},\\ \overline{IP}_3(A) &= \underline{IP}_3(A)\bigcup\{IMd_{IP}(a):a\in A-\underline{IP}_3(A)\}.\end{aligned}$$

A subset $A\subseteq E$ is called third type of inverse soft covering based definable if $\overline{IP}_3(A)=\underline{IP}_3(A)$; in the opposite case, that is, if $\overline{IP}_3(A)\neq\underline{IP}_3(A)$, A is said to be third type of inverse soft covering based rough set.

Before defining the new type of inverse soft covering based sets, let us give a remark about the concept of IMd_{IP}.

Remark 1. Let $IP=(E,IC_S)$ be an inverse soft covering approximation space. Then for all $A\subseteq E$,

$$\bigcup\{IMd_{IP}(a):a\in A\}\neq\bigcup\{\bigcup IMd_{IP}(a):a\in A\}.$$

Example 3.1. Let $X=\{x_1,x_2,x_3\}$ be a universe and $S=(\Lambda,X)$ an inverse soft covering set over X, where $E=\{e_1,e_2,e_3,e_4\}$, $\Lambda(x_1)=\{e_1,e_2\}$, $\Lambda(x_2)=\{e_2,e_3\}$, $\Lambda(x_3)=\{e_3,e_4\}$. Then $IP=(E,IC_S)$ is an inverse soft covering approximation space.

For $A_1=\{e_2,e_3\}\subseteq E$, it is easy to check that

$$IMd_{IP}(e_2)=\{\Lambda(x_1),\Lambda(x_2)\},IMd_{IP}(e_3)=\{\Lambda(x_2),\Lambda(x_3)\}.$$

We have that

$$\begin{aligned}\bigcup\{IMd_{IP}(a):a\in A_1\} &= \bigcup\{IMd_{IP}(e_2),IMd_{IP}(e_3)\}\\ &= \bigcup\{\{\Lambda(x_1),\Lambda(x_2)\},\{\Lambda(x_2),\Lambda(x_3)\}\}\\ &= \{\Lambda(x_1),\Lambda(x_2),\Lambda(x_3)\}\end{aligned}$$

and

$$\bigcup\{\bigcup IMd_{IP}(a):a\in A_1\}=\bigcup\{\{\Lambda(x_1),\Lambda(x_2),\Lambda(x_3)\}\}=\{e_1,e_2,e_3,e_4\}.$$

Hence we obtain that $\bigcup\{IMd_{IP}(a):a\in A_1\}\neq\bigcup\{\bigcup IMd_{IP}(a):a\in A_1\}$.

Remark 2. It is easy to see from the example above we obtain that $\bigcup\{IMd_{IP}(a):a\in A_1\}\subseteq P(E)$ and so $\bigcup\{IMd_{IP}(a):a\in A_1\}\notin P(E)$. Also we obtain that $\bigcup\{\bigcup IMd_{IP}(a):a\in A_1\}\in P(E)$. According to the result we obtained from the example above, we revise and rewrite the Definition 3.6 and Definition 3.7 by writing $\bigcup\{\bigcup IMd_{IP}(a):a\in A\}$ instead of $\bigcup\{IMd_{IP}(a):a\in A\}$.

Definition 3.8. Let $S=(\Lambda,X)$ be an inverse soft covering set over X and $IP=(E,IC_S)$ an inverse soft covering approximation space. For a set $A\subseteq E$, the inverse soft covering lower and upper approximations are, respectively, defined as

$$\begin{aligned}\underline{IP}_2(A) &= \bigcup_{x\in X}\{\Lambda(x):\Lambda(x)\subseteq A\},\\ \overline{IP}_2(A) &= \bigcup\{\bigcup IMd_{IP}(a):a\in A\}.\end{aligned}$$

A subset $A\subseteq E$ is called second type of inverse soft covering based definable if $\overline{IP}_2(A)=\underline{IP}_2(A)$; in the opposite case, that is, if $\overline{IP}_2(A)\neq\underline{IP}_2(A)$, A is said to be second type of inverse soft covering based rough set.

Definition 3.9. Let $S=(\Lambda,X)$ be an inverse soft covering set over X and $IP=(E,IC_S)$ an inverse soft covering approximation space. For a set $A\subseteq E$, the inverse soft covering lower and upper approximations are, respectively, defined as

$$\begin{aligned}\underline{IP}_3(A) &= \bigcup_{x\in X}\{\Lambda(x):\Lambda(x)\subseteq A\},\\ \overline{IP}_3(A) &= \underline{IP}_3(A)\bigcup\{\bigcup\{\bigcup IMd_{IP}(a):a\in A-\underline{IP}_3(A)\}\}.\end{aligned}$$

A subset $A\subseteq E$ is called third type of inverse soft covering based definable if $\overline{IP}_3(A)=\underline{IP}_3(A)$; in the opposite case, that is, if $\overline{IP}_3(A)\neq\underline{IP}_3(A)$, A is said to be third type of inverse soft covering based rough set.

4 New type of inverse soft covering based rough sets

In this section, we present a new kind of inverse soft covering based rough sets and its basic properties. We present the similarity and difference between some fundamental properties of this type of inverse soft covering based rough sets and those of Pawlak's rough sets [2]. Pawlak showed that lower and upper approximation operations on a set are the interior and closure operators on topological spaces, respectively [2]. This result allowed the establishment of a relationship between topological spaces and approximation spaces and became the starting point for the study of topological concepts in rough set theory. In this paper, we use these topological tools to investigate a new type of soft covering based rough sets. The new type of inverse soft covering based rough set model as follows:

Definition 4.1. Let $S=(\Lambda,X)$ be an inverse soft covering set over X and $IP=(E,IC_S)$ an inverse soft covering approximation space. For a set $A\subseteq E$, the fourth type of inverse soft covering lower and upper approximation are, respectively, defined as

$$\begin{aligned} \underline{IP}_4(A) &= \bigcup_{x\in X}\{\Lambda(x):\Lambda(x)\subseteq A\}, \\ \overline{IP}_4(A) &= \underline{IP}_4(A)\bigcup(\bigcup\{\bigcap_{\Lambda(x)\in IMd_{IP}(a)}\Lambda(x): a\in A-\underline{IP}_4(A)\}). \end{aligned}$$

In addition,

$$\begin{aligned} IPos_{IP}(A) &= \underline{IP}_4(A), \\ INeg_{IP}(A) &= E-\overline{IP}_4(A), \\ IBnd_{IP}(A) &= \overline{IP}_4(A)-\underline{IP}_4(A) \end{aligned}$$

are called the fourth type of inverse soft covering positive, negative and boundary regions of A, respectively.

Definition 4.2. Let $S=(\Lambda,X)$ be an inverse soft covering set over X and $IP=(E,IC_S)$ an inverse soft covering approximation space. A subset $A\subseteq E$ is called fourth type of inverse soft covering based definable if $\overline{IP}_4(A)=\underline{IP}_4(A)$; in the opposite case, that is, if $\overline{IP}_4(A)\neq\underline{IP}_4(A)$, A is said to be fourth type of inverse soft covering based rough set.

Example 4.1. Let $X=\{x_1,x_2,x_3,x_4\}$ be a universe and $S=(\Lambda,X)$ an inverse soft covering set over X, where $E=\{e_1,e_2,e_3,e_4,e_5,e_6\}$, $\Lambda(x_1)=\{e_1,e_2,e_3\}$, $\Lambda(x_2)=\{e_1,e_2,e_4,e_5\}$, $\Lambda(x_3)=\{e_2,e_3,e_4,e_5\}$ and $\Lambda(x_4)=\{e_3,e_4,e_5,e_6\}$. Then $IP=(E,IC_S)$ is an inverse soft covering approximation space.

For $A_1=\{e_1,e_2,e_3,e_4,e_5\}\subseteq E$, we have $\underline{IP}_4(A_1)=\{e_1,e_2,e_3,e_4,e_5\}$, $\overline{IP}_4(A_1)=\{e_1,e_2,e_3,e_4,e_5\}$. Thus, $\underline{IP}_4(A_1)=\overline{IP}_4(A_1)$ and A_1 is fourth type of inverse soft covering based definable set.

For $A_2=\{e_1,e_3,e_4,e_5,e_6\}\subseteq E$, we have $\underline{IP}_4(A_2)=\{e_3,e_4,e_5,e_6\}$, $\overline{IP}_4(A_2)=\{e_1,e_2,e_3,e_4,e_5,e_6\}=E$. Thus, $\underline{IP}_4(A_2)\neq\overline{IP}_4(A_2)$ and A_2 is fourth type of inverse soft covering based rough set.

Remark 3. It is easy to see from the definitions that the fourth type of inverse soft covering lower approximation is the same as those in above three types of inverse soft covering based rough set model.

Comparing with the properties of classical rough sets, the inverse soft covering lower approximation has the following properties:

Theorem 4.1. *[19] Let $S=(\Lambda,X)$ be an inverse soft covering set over X, $IP=(E,IC_S)$ an inverse soft covering approximation space and A, $B\subseteq E$. Then the inverse soft covering lower approximation has the following properties:*

1. $\underline{IP}_4(E)=E$
2. $\underline{IP}_4(\emptyset)=\emptyset$
3. $\underline{IP}_4(A)\subseteq A\ for\ all\ A\subseteq E$
4. $A\subseteq B\Rightarrow\underline{IP}_4(A)\subseteq\underline{IP}_4(B)$
5. $\underline{IP}_4(\underline{IP}_4(A))=\underline{IP}_4(A)$
6. $\forall x\in X$, $\underline{IP}_4(\Lambda(x))=\Lambda(x)$
7. $\underline{IP}_4(A\cap B)\subseteq\underline{IP}_4(A)\cap\underline{IP}_4(B)$
8. $\underline{IP}_4(A\cup B)\supseteq\underline{IP}_4(A)\cup\underline{IP}_4(B)$.

Now, we present another representation of the fourth type of inverse soft covering upper approximation.

Theorem 4.2. *Let $S=(\Lambda,X)$ be an inverse soft covering set over X, $IP=(E,IC_S)$ an inverse soft covering approximation space and $A\subseteq E$. Then*

$$\overline{IP}_4(A)=\bigcup\{\bigcap_{\Lambda(x)\in IMd_{IP}(a)}\Lambda(x):x\in X,\ a\in A\}.$$

Proof. If $\underline{IP}_4(A)=\emptyset$, from Definition 4.1, this is obvious. If $\underline{IP}_4(A)\neq\emptyset$, then

$$\cup\{\bigcap_{\Lambda(x)\in IMd_{IP}(a)}\Lambda(x):a\in A\}=(\cup\{\bigcap_{\Lambda(x)\in IMd_{IP}(a)}\Lambda(x):a\in\underline{IP}_4(A)\})\cup(\cup\{\bigcap_{\Lambda(x)\in IMd_{IP}(a)}\Lambda(x):a\in A-\underline{IP}_4(A)\}).$$

So we only need to prove that $\cup\{\bigcap_{\Lambda(x)\in IMd_{IP}(a)}\Lambda(x):a\in\underline{IP}_4(A)\}=\underline{IP}_4(A)$. From Definition 3.4, for all $x\in X$, each $\Lambda(x)\subseteq A$, we have $\cup\{\bigcap_{\Lambda'(x)\in IMd_{IP}(a)}\Lambda'(x):a\in\Lambda(x)\}=\Lambda(x)$, so

$$\underline{IP}_4(A)=\bigcup_{x\in X}\{\Lambda(x):\Lambda(x)\subseteq A\}=\bigcup_{x\in X}(\cup\{\bigcap_{\Lambda'(x)\in IMd_{IP}(a)}\Lambda'(x):a\in\Lambda(x)\subseteq A\}).$$

Since for all $x\in X$, each $\Lambda(x)\subseteq A$, $\Lambda(x)\subseteq\underline{IP}_4(A)$, we obtain $\underline{IP}_4(A)=\cup\{\bigcap_{\Lambda'(x)\in IMd_{IP}(a)}\Lambda'(x):a\in\underline{IP}_4(A)\}$. This shows that $\overline{IP}_4(A)=\bigcup\{\bigcap_{\Lambda(x)\in IMd_{IP}(a)}\Lambda(x):x\in X,a\in A\}$.

Now, we investigate some properties of the fourth type of inverse soft covering upper approximation.

Theorem 4.3. *Let $S=(\Lambda,X)$ be an inverse soft covering set over X, $IP=(E,IC_S)$ an inverse soft covering approximation space and A, $B\subseteq E$. Then the fourth type of inverse soft covering upper approximation has the following properties:*

1. $\overline{IP}_4(E)=E$
2. $\overline{IP}_4(\emptyset)=\emptyset$
3. $A\subseteq\overline{IP}_4(A)$
4. $\overline{IP}_4(A\cup B)=\overline{IP}_4(A)\cup\overline{IP}_4(B)$
5. $\overline{IP}_4(\overline{IP}_4(A))=\overline{IP}_4(A)$
6. $A\subseteq B\Rightarrow\overline{IP}_4(A)\subseteq\overline{IP}_4(B)$
7. $\forall x\in X$, $\overline{IP}_4(\Lambda(x))=\Lambda(x)$.

Proof. From Definition 4.1 we can easily prove that properties 1,2 and 3.

4) From Definition 4.1 and Theorem 4.2 we have

$$\overline{IP}_4(A)=\cup\{\bigcap_{\Lambda(x)\in IMd_{IP}(a)}\Lambda(x):x\in X,a\in A\}$$

and

$$\overline{IP}_4(B)=\cup\{\bigcap_{\Lambda(x)\in IMd_{IP}(a)}\Lambda(x):x\in X,a\in B\}$$

for all A, $B\subseteq E$. So

$$\begin{aligned}\overline{IP}_4(A)\cup\overline{IP}_4(B) &= (\cup\{\bigcap_{\Lambda(x)\in IMd_{IP}(a)}\Lambda(x):x\in X,a\in A\})\cup(\cup\{\bigcap_{\Lambda(x)\in IMd_{IP}(a)}\Lambda(x):x\in X,a\in B\})\\ &= \cup\{\bigcap_{\Lambda(x)\in IMd_{IP}(a)}\Lambda(x):a\in A\cup B\}=\overline{IP}_4(A\cup B).\end{aligned}$$

5) From the property $A\subseteq\overline{IP}_4(A)$, we have $\overline{IP}_4(A)\subseteq\overline{IP}_4(\overline{IP}_4(A))$. $\forall a\in\overline{IP}_4(\overline{IP}_4(A))$, from Definition 4.1 and Theorem 4.2, we have

$$\overline{IP}_4(\overline{IP}_4(A))=\cup\{\bigcap_{\Lambda'(x)\in IMd_{IP}(y)}\Lambda'(x):y\in\overline{IP}_4(A)\}.$$

Hence there exists some

$$y_0\in\overline{IP}_4(A)=\cup\{\bigcap_{\Lambda(x)\in IMd_{IP}(z)}\Lambda(x):z\in A\},\ \text{and } a\in\bigcap_{\Lambda(x)'\in IMd_{IP}(y_0)}\Lambda'(x).$$

Then there exists $z_0\in A$ such that $y_0\in\bigcap_{\Lambda(x)\in IMd_{IP}(z_0)}\Lambda(x)$. So for all $\Lambda(x)\in IMd_{IP}(z_0)$, we have $y_0\in\Lambda(x)$. Therefore for every such $\Lambda(x)$, there must exist some $\Lambda'(x)\in IMd_{IP}(y_0)$ satisfying $\Lambda'(x)\subseteq\Lambda(x)$. Thus we obtain

$$\bigcap_{\Lambda(x)'\in IMd_{IP}(y_0)}\Lambda'(x)\subseteq\bigcap_{\Lambda(x)\in IMd_{IP}(z_0)}\Lambda(x),\ \text{so } a\in\bigcap_{\Lambda(x)\in IMd_{IP}(z_0)}\Lambda(x).$$

Therefore, $\overline{IP}_4(\overline{IP}_4(A))=\overline{IP}_4(A)$.

6) If $A\subseteq B$, from Theorem 4.2, $\overline{IP}_4(A)=\cup\{\bigcap_{\Lambda(x)\in IMd_{IP}(a)}\Lambda(x):a\in A\}\subseteq\cup\{\bigcap_{\Lambda(x)\in IMd_{IP}(a)}\Lambda(x):a\in B\}=\overline{IP}_4(B)$.

7) If for all $x\in X$, then $\underline{IP}_4(\Lambda(x))=\Lambda(x)$. Thus from Definition 4.1, $\overline{IP}_4(\Lambda(x))=\Lambda(x)$.

Remark 4. Let $S=(\Lambda,X)$ be an inverse soft covering set over X, $IP=(E,IC_S)$ an inverse soft covering approximation space and $A,B\subseteq E$. Then the fourth type of inverse soft covering lower and upper approximations do not have the following properties:

1. $\underline{IP}_4(A\cap B)=\underline{IP}_4(A)\cap\underline{IP}_4(B)$
2. $\underline{IP}_4\left(-\underline{IP}_4(A)\right)=-\underline{IP}_4(A)$
3. $\overline{IP}_4(-\overline{IP}_4(A))=-\overline{IP}_4(A)$
4. $\underline{IP}_4(A)=-\overline{IP}_4(-A)$
5. $\overline{IP}_4(A)=-\underline{IP}_4(-A)$

The following examples show that the equalities mentioned above do not hold.

Example 4.2. Let $X=\{x_1,x_2,x_3,x_4,x_5\}$ be a universe and $S=(\Lambda,X)$ an inverse soft covering set over X, where $E=\{e_1,e_2,e_3,e_4,e_5,e_6,e_7\}$, $\Lambda(x_1)=\{e_1,e_7\}$, $\Lambda(x_2)=\{e_2,e_5\}$, $\Lambda(x_3)=\{e_3,e_4,e_6\}$, $\Lambda\left(x_4\right)=\{e_5,e_6\}$ and $\Lambda(x_5)=\{e_2,e_3\}$. Then $IP=(E,IC_S)$ is an inverse soft covering approximation space.

1. Suppose that $A=\{e_2,e_3\}\subseteq E$ and $B=\{e_2,e_5\}\subseteq E$. $\underline{IP}_4(A)=\{e_2,e_3\}$, $\underline{IP}_4(B)=\{e_2,e_5\}$, $\underline{IP}_4(A)\cap\underline{IP}_4(B)=\{e_2\}$ and $\underline{IP}_4(A\cap B)=\emptyset$. This shows that $\underline{IP}_4(A\cap B)\neq\underline{IP}_4(A)\cap\underline{IP}_4(B)$.
2. Suppose that $A=\{e_5,e_6\}\subseteq E$. $\underline{IP}_4(A)=\{e_5,e_6\}$, $-\underline{IP}_4(A)=\{e_1,e_2,e_3,e_4,e_7\}$, $\underline{IP}_4\left(-\underline{IP}_4(A)\right)=\{e_1,e_2,e_3,e_7\}$. This shows that $\underline{IP}_4\left(-\underline{IP}_4(A)\right)\neq-\underline{IP}_4(A)$.

Example 4.3. Let $X=\{x_1,x_2,x_3\}$ be a universe and $S=(\Lambda,X)$ an inverse soft covering set over X, where $E=\{e_1,e_2,e_3,e_4,e_5,e_6\}$, $\Lambda(x_1)=\{e_1,e_2\}$, $\Lambda(x_2)=\{e_1,e_2,e_3\}$ and $\Lambda(e_3)=\{e_3,e_4,e_5,e_6\}$. Then $IP=(E,IC_S)$ is an inverse soft covering approximation space. Suppose that $A=\{e_1,e_2,e_3\}\subseteq E$.

1. $\overline{IP}_4(A)=\{e_1,e_2,e_3\}$, $-\overline{IP}_4(A)=\{e_4,e_5,e_6\}$, $\overline{IP}_4(-\overline{IP}_4(A))=\{e_3,e_4,e_5,e_6\}$. This shows that $\overline{IP}_4(-\overline{IP}_4(A))\neq-\overline{IP}_4(A)$.
2. $\underline{IP}_4(A)=\{e_1,e_2,e_3\}$, $\overline{IP}_4(-A)=\{e_3,e_4,e_5,e_6\}$, $-\overline{IP}_4(-A)=\{e_1,e_2\}$. This shows that $\underline{IP}_4(A)\neq-\overline{IP}_4(-A)$.
3. $\overline{IP}_4(A)=\{e_1,e_2,e_3\}$, $\underline{IP}_4(-A)=\emptyset$, $-\underline{IP}_4(-A)=E=\{e_1,e_2,e_3,e_4,e_5,e_6\}$. This shows that $\overline{IP}_4(A)\neq-\underline{IP}_4(-A)$.

5 Comparison of four types of inverse soft covering upper approximations

In literature, we already have three types of inverse soft covering based rough sets. What are the relationships between the fourth one and the existed three ones? From the definitions of four types of inverse soft covering upper approximation operations, we have the following relationships.

$$\overline{IP}_4(A)\subseteq\overline{IP}_3(A)\subseteq\overline{IP}_2(A)\subseteq\overline{IP}_1(A)\text{ for all }A\subseteq E.$$

It is evident that the fourth inverse soft covering upper aproximation has the more exactitude than others have.

Remark 5. $\overline{IP}_1(A)\subset\overline{IP}_2(A)\subset\overline{IP}_3(A)\subset\overline{IP}_4(A)$ is not true in general as shown in the following example.

Example 5.1. Let $X=\{x_1,x_2,x_3,x_4,x_5\}$ be a universe set and $S=(\Lambda,X)$ an inverse soft covering set over X, where $E=\{e_1,e_2,e_3,e_4,e_5,e_6\}$, $\Lambda(x_1)=\{e_1,e_3\}$, $\Lambda(x_2)=\{e_2,e_4\}$, $\Lambda(x_3)=\{e_2,e_5\}$, $\Lambda\left(x_4\right)=\{e_1,e_3,e_4\}$ and $\Lambda(x_5)=\{e_2,e_4,e_6\}$. Then $IP=(E,IC_S)$ is an inverse soft covering approximation space. For $A=\{e_2,e_4\}\subseteq E$, we have

$$\begin{aligned}\overline{IP}_3(A)&=\underline{IP}_3(A)\bigcup\{\bigcup\{\bigcup IMd_{IP}(a):a\in A-\underline{IP}_3(A)\}\}=\{e_2,e_4\},\\ \overline{IP}_2(A)&=\bigcup\{\bigcup IMd_{IP}(a):a\in A\}=\{e_1,e_2,e_3,e_4,e_5\},\\ \overline{IP}_1(A)&=\bigcup_{x\in X}\{\Lambda(x):\Lambda(x)\cap A\neq\emptyset\}=E.\end{aligned}$$

Thus, we obtain $\overline{IP}_1(A)\not\subset\overline{IP}_2(A)\not\subset\overline{IP}_3(A)$.

Example 5.2. Let $X=\{x_1,x_2,x_3\}$ be a universe set and $S=(\Lambda,X)$ an inverse soft covering set over X, where $E=\{e_1,e_2,e_3,e_4\}$, $\Lambda(x_1)=\{e_1,e_2\}$, $\Lambda(x_2)=\{e_2,e_3\}$ and $\Lambda(x_3)=\{e_2,e_3,e_4\}$. Then $IP=(E,IC_S)$ is an inverse soft covering approximation space. For $A=\{e_2,e_4\}\subseteq E$, we have

$$\begin{aligned}\overline{IP}_4(A)&=\underline{IP}_4(A)\bigcup(\bigcup\{\bigcap_{F(x)\in IMd_{IP}(a)}F(x):a\in A-\underline{IP}_4(A)\})=\{e_2,e_3,e_4\},\\ \overline{IP}_3(A)&=\underline{IP}_3(A)\bigcup\{\bigcup\{\bigcup IMd_{IP}(a):a\in A-\underline{IP}_3(A)\}\}=E.\end{aligned}$$

Thus, we obtain $\overline{IP}_3(A)\not\subset\overline{IP}_4(A)$.

6 An application of multicriteria group decision making

Many researchers have produced many decision making methods techniques by means of very different modelling approaches such as fuzzy set theory, rough set theory, soft set theory, and their hybrid models for a variety of settings. In 2016, Cetkin et al. [18] introduced the concept of inverse soft set theory, offering a new approach to decision-making problems by applying inverse soft sets instead of soft sets. Inspired by this work, in this section, we present a new method based on the decision-making method proposed by Feng in 2011 [11]. We redesigne some steps of Feng's method. Also we inverse soft covering based rough sets instead of soft rough sets. As a result, we develop an algorithm and present an example which shows that a decision making method can be successfully applied to many problems that contain uncertainties. We use inverse soft covering upper and lower approximations to support this decision making process. This method is characterized by our inverse soft covering upper approximation. In the previous section we showed that our inverse soft covering upper approximation is smaller than with other types of inverse soft covering upper approximations. Therefore the boundaries are reduced by means of new approximation, the definability increases. It is evident that our oft multi upper approximation has the more exactitude than the other ones. Making it a valuable addition to the field. Thus we may expect to gain much more useful information with the help of our inverse soft covering upper approximation.

The purpose of this method is to improve the primary evaluation of the whole expert group and allow us to more reliably select the most suitable criterion of the objects. In the proposed schemes, consider X be the objects under observation, E be the set of criterions the objects in X. We take an inverse soft covering set $S=(\Lambda, X)$ for real worlds problems. Consider $Z=\{D_1, D_2, ..., D_n\}$ be the set consisting to decision makers who examine the criterions to identify the possible solution and A_i be the initial estimation derived by members of experts D_i which is express by the inverse soft set $\Omega=(\omega, Z)$. To get better results we find out inverse soft covering upper and lower approximations of initial estimated results A_i according to inverse soft covering approximation space $IP=(E, IC_S)$, consequently we obtain inverse soft sets $\Omega_*=(\underline{\omega}_*, Z)$ and $\Omega^*=(\overline{\omega}^*, Z)$. The inverse soft covering lower approximation can be interpreted as the set consisting of the criterions which are certainly the optimum candidates according to expert D_i's evaluation. Similarly, the inverse soft covering upper approximation can be interpreted as the set consisting of the criterions which are possibly the optimum candidates according to expert D_i's evaluation. Following these inverse soft sets, we define inverse fuzzy soft sets μ_{Ω_*}, μ_{Ω} and μ_{Ω^*}. We compute decision value

$$\mu_{\Omega_*+\Omega+\Omega_*}(e_k)=\mu_{\Omega_*}(e_k)+\mu_{\Omega}(e_k)+\mu_{\Omega^*}(e_k)-\left[\mu_{\Omega_*}(e_k)\times\mu_{\Omega}(e_k)\times\mu_{\Omega^*}(e_k)\right]$$

of each alternative $e_k \in E$. Then rank all the alternatives according to their decision values; one can select any of the criterions with the largest decision value as the most preferred alternative. Now, we will present the notations, steps and the algorithm for the new approach, respectively, in detail.

Table 1: Group Decision-Making Algorithm Using Symbols

Symbols	Meaning	Notes
$X=\{x_1, x_2, .., x_n\}$	Alternatives	The set of objects to be evaluated
$E=\{e_1, e_2, .., e_m\}$	Parameters	All criteria considered for evaluation
$S=(\Lambda, X)$	Original description inverse soft covering set	$S=(\Lambda, X)$ describes the given data
$IP=(E, IC_S)$	Inverse soft covering approximation space	$IP=(E, IC_S)$ an inverse soft covering approximation space create from $S=(\Lambda, X)$
$Z=\{D_1, D_2, ..., D_n\}$	Expert group	Each expert evaluates all alternatives and selects optimal alternatives
A_i	Primary evaluation result of specialist expert D_i	The set of alternatives that the specialist considers optimal
$\Omega=(\omega, Z)$	Evaluation inverse soft set of the expert group Z	$\Omega=(\omega, Z)$ describes the primary evaluation results of the expert group Z
$\Omega_*=(\underline{\omega}_*, Z)$	Inverse soft set	Inverse soft covering lower approximations of A_i according to $IP=(E, IC_S)$
$\Omega^*=(\overline{\omega}^*, Z)$	Inverse soft set	Inverse soft covering upper approximations of A_i according to $IP=(E, IC_S)$
μ_{Ω}	Inverse fuzzy soft set	μ_{Ω} corresponding to the inverse soft set $\Omega=(\omega, Z)$
μ_{Ω_*}	Inverse fuzzy soft set	μ_{Ω_*} corresponding to the inverse soft set $\Omega_*=(\underline{\omega}_*, Z)$
μ_{Ω^*}	Inverse fuzzy soft set	μ_{Ω^*} corresponding to the inverse soft set $\Omega^*=(\overline{\omega}^*, Z)$

ALGORITHM

1. Step: Write the original description inverse soft covering set $S=(\Lambda, X)$ which describes the given data.

2. Step: Consider a group of decision makers $Z=\{D_1, D_2, ..., D_n\}$ and construct the evaluation inverse soft set $\Omega=(\omega, Z)$ using the primary evaluation results of the expert group Z.

3. Step: Obtain inverse soft covering rough approximations in the form of inverse soft sets $\Omega_*=(\underline{\omega}_*, Z)$ and $\Omega^*=(\overline{\omega}^*, Z)$.

4. Step: Define inverse fuzzy soft set μ_{Ω_*}, μ_{Ω} and μ_{Ω^*} corresponding to the inverse soft sets $\Omega_*=(\underline{\omega}_*, Z), \Omega=(\omega, Z)$ and $\Omega^*=(\overline{\omega}^*, Z)$ defined by the formulas:

$$\mu_{\Omega_*}(e_k) = \frac{1}{n}\sum_{i=1}^{n} C_{\underline{\omega}_* D_i}(e_k)$$
$$\mu_{\Omega}(e_k) = \frac{1}{n}\sum_{i=1}^{n} C_{\omega D_i}(e_k)$$
$$\mu_{\Omega^*}(e_k) = \frac{1}{n}\sum_{i=1}^{n} C_{\overline{\omega}^* D_i}(e_k)$$

5. Step: Find the final decision set by adding $\Omega_* = (\underline{\omega}_*, Z), \Omega = (\omega, Z)$ and $\Omega^* = (\overline{\omega}^*, Z)$ calculated as

$$\mu_{\Omega_*+\Omega+\Omega_*}(e_k) = \mu_{\Omega_*}(e_k) + \mu_{\Omega}(e_k) + \mu_{\Omega^*}(e_k) - \left[\mu_{\Omega_*}(e_k) \times \mu_{\Omega}(e_k) \times \mu_{\Omega^*}(e_k)\right]$$

6. Step: Finally the alternative having maximum decision value can be selected as optimal solution.

Now, we propose a concrete example concerning the selection of symptoms which has more chances of depression by the psychiatrists. Assume that there is a set of 4 patients with a set of 5 symptoms related to a set of patients suffering from depression.

Example 6.1. Let $X = \{x_1, x_2, x_3, x_4\}$ be a universe set which is the collection of patients suffering from depression, $E = \{e_1, e_2, e_3, e_4, e_5\}$ be a parameter set which is the collection of factors affecting depression and $S = (\Lambda, X)$ an inverse soft covering set over X, where $\Lambda(x_1) = \{e_1, e_2\}$, $\Lambda(x_2) = \{e_1, e_3\}$, $\Lambda(x_3) = \{e_1, e_3, e_4\}$ and $\Lambda\left(x_4\right) = \{e_1, e_3, e_4, e_5\}$. Then $IP = (E, IC_S)$ is an inverse soft covering approximation space.

Now, we show how to use a new type of inverse soft covering upper and lower approximations to support this group decision making process. Let us determine the most important factor affecting depression.

1. Step: Write the original description inverse soft covering set $S = (\Lambda, X)$ which describes the given data. Since $S = (\Lambda, X)$ be an inverse soft covering set over X, then $IP = (E, IC_S)$ is an inverse soft covering approximation space.

2. Step: Consider a group of decision makers $Z = \{D_1, D_2, ..., D_n\}$ and construct the evaluation inverse soft set $\Omega = (\omega, Z)$ using the primary evaluation results of the expert group Z. For simplicity, we assume that the evaluations of these specialists in $Z = \{D_1, D_2, D_3\}$ are of the same importance. $Z = \{D_1, D_2, D_3\}$ are the specialist psychiatrists group who analyze the factors affecting depression depending upon related parameters. A_i is the initial assessment result of the psychiatrists. Each psychiatrists has different opinions about the factors affecting depression. Each expert identifies the three most important factors affecting depression. Now we generate the inverse soft set $\Omega = (\omega, Z)$ primary evaluation result of experts are

$$\begin{aligned}
A_1 &= \omega(D_1) = \{e_1, e_3, e_4\}, \\
A_2 &= \omega(D_2) = \{e_1, e_2, e_3\}, \\
A_3 &= \omega(D_3) = \{e_1, e_2, e_5\}.
\end{aligned}$$

3. Step: Obtain inverse soft covering rough approximations in the form of inverse soft sets $\Omega_* = (\underline{\omega}_*, Z)$ and $\Omega^* = (\overline{\omega}^*, Z)$. Now, we use a new type of inverse soft covering upper and lower approximations

$$\begin{aligned}
\underline{\omega}_*(D_1) &= \underline{IP}_4(A_1) = \{e_1, e_3, e_4\}, \\
\underline{\omega}_*(D_2) &= \underline{IP}_4(A_2) = \{e_1, e_2, e_3\}, \\
\underline{\omega}_*(D_3) &= \underline{IP}_4(A_3) = \{e_1, e_2\}, \\
\overline{\omega}^*(D_1) &= \overline{IP}_4(A_1) = \{e_1, e_3, e_4\}, \\
\overline{\omega}^*(D_2) &= \overline{IP}_4(A_2) = \{e_1, e_2, e_3\}, \\
\overline{\omega}^*(D_3) &= \overline{IP}_4(A_3) = E = \{e_1, e_2, e_3, e_4, e_5\}.
\end{aligned}$$

Following these inverse soft covering rough approximations, we get two inverse soft sets $\Omega_* = (\underline{\omega}_*, Z)$ and $\Omega^* = (\overline{\omega}^*, Z)$ where, $\underline{\omega}_*(D_i) = \underline{IP}_4(A_i)$ and $\overline{\omega}^*(D_i) = \overline{IP}_4(A_i)$.

4. Step: Now, we define inverse fuzzy soft set μ_{Ω_*}, μ_{Ω} and μ_{Ω^*} corresponding to the inverse soft sets $\Omega_* = (\underline{\omega}_*, Z), \Omega = (\omega, Z)$ and $\Omega^* = (\overline{\omega}^*, Z)$ defined by the formulas:

$$\mu_{\Omega_*}(e_k) = \frac{1}{3}\sum_{i=1}^{3} C_{\underline{\omega}_* D_i}(e_k),$$
$$\mu_{\Omega}(e_k) = \frac{1}{3}\sum_{i=1}^{3} C_{\omega D_i}(e_k),$$
$$\mu_{\Omega^*}(e_k) = \frac{1}{3}\sum_{i=1}^{3} C_{\overline{\omega}^* D_i}(e_k).$$

Thus, we have

$$\begin{aligned}
\mu_{\Omega_*}(e_k) &= \{(e_1,1),(e_2,\frac{2}{3}),(e_3,\frac{2}{3}),(e_4,\frac{1}{3}),(e_5,0)\} \\
\mu_{\Omega}(e_k) &= \{(e_1,1),(e_2,\frac{2}{3}),(e_3,\frac{2}{3}),(e_4,\frac{1}{3}),(e_5,\frac{1}{3})\} \\
\mu_{\Omega^*}(e_k) &= \{(e_1,1),(e_2,\frac{2}{3}),(e_3,1),(e_4,\frac{2}{3}),(e_5,\frac{1}{3})\}.
\end{aligned}$$

5. Step: We find the final decision set by adding $\Omega_* = (\underline{\omega}_*, Z), \Omega = (\omega, Z)$ and $\Omega^* = (\overline{\omega}^*, Z)$ calculated as

$$\mu_{\Omega_*+\Omega+\Omega_*}(e_k) = \mu_{\Omega_*}(e_k) + \mu_{\Omega}(e_k) + \mu_{\Omega^*}(e_k) - \left[\mu_{\Omega_*}(e_k) \times \mu_{\Omega}(e_k) \times \mu_{\Omega^*}(e_k)\right]$$

Then we can arrange the criterions according to their decision values as:

$$\begin{array}{ccccc}
e_1 & > e_3 & > e_2 & > e_4 & > e_5 \\
2 & > 1,889 & > 1,704 & > 1,259 & > 0,66
\end{array}$$

6. Step: Finally the alternative having maximum decision value can be selected as optimal solution. Since e_1 is the factor having maximum decision value. e_1 is selected by the psychiatrists which is the most important factor affecting depression.

Remark 6. In this study, it is aimed to make the task of determining the most suitable choice easier by using new approximation. In this models we can notice that the use of inverse soft covering rough approximations procedure refines the primary evaluation results. In the proposed schemes, each expert simply gives an initial set consisting of the preferable alternatives in the corresponding expert's point of view. The primary evaluation results of the expert group store in the evaluation inverse soft set and then approximate in the original description inverse soft set using inverse soft covering upper and lower approximations. Finally, all the obtain data of the whole expert group can be synthesized into an inverse fuzzy soft set and all the alternatives will be ranked according to their decision values. It should be noted that the use of our inverse soft covering upper and lower approximations could, to some extent, automatically reduce the noise factor caused by the subjective nature of the expert's evaluation. Because when our approaches are used, the boundaries of the primary evaluation results of the whole expert group become smaller and their definability increases. This makes the results is more realistic. This makes the decision making to be more preferable to reflect the reality.

7 Conclusion

Inverse soft covering based rough set describes the roughness of inverse soft set. Three types of inverse soft covering based rough sets have been studied in literature. In this paper, we proposed a new approach in developing inverse soft covering based rough sets using a new type of inverse soft covering upper approximation. Also we investigated the relationships between this type of inverse soft covering based rough sets and the existing ones given by Demirtaş et al. [19]. While the lower approximation in our model remains consistent with the existing three types, the upper approximation differs significantly, offering a more precise and accurate approach compared to the previously presented types. We applied it to a decision-making problem to demonstrate the applicability of the proposed method. We obtained much more useful information with the help of our inverse soft covering approximations and to reduce possible error to some extent caused by personal nature of analyst in the decision-making results.

This study can be seen as a source of motivation for many researchers since the decision-making processes managed by new type of inverse soft covering approximations, especially for the uncertainty problems, in a more ideal way.

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