

András Kroó: a life in approximation theory

Stefano De Marchi^a · Marco Vianello^a

Abstract

We briefly describe the most important contributions made by András Kroó to approximation theory during the first 65 years of his life as well as some less well-known facts.

1 András Kroó's scientific contributions: a brief summary

András Kroó was born on the 23rd of August, 1954, in a small city in eastern Hungary, Uzsgorod, which at that time was part of the Soviet Union. From 1971 to 1976 he studied Mathematics at Moscow State University, graduating with a Master's degree, and then at the Mathematical Institute, Hungarian Academy of Sciences, where he received his PhD in 1979.

His first paper was published in 1977, was written by himself in Russian, and concerned the correctness of best trigonometric approximation. For about 8 years he studied various other aspects of approximation by polynomials in different function spaces, including in the complex plane. In 1989, together with Paul Erdős and his present colleague Josef Szabados, they published a paper in which they study the convergence of interpolating polynomials. Consequently his *Erdős number* is 1!

The first part of his contributions to approximation theory, which consists of about 130 papers, were mostly devoted to understanding different aspects of best approximation by polynomials and trigonometric functions, also in different function spaces (papers up to '90). He then started to investigate the problem of density of polynomial spaces, mainly in the multivariate setting, proving various interesting results concerning extremal points of generalized Chebyshev polynomials. What is worth noting is that he worked mainly alone. He studied also families of orthogonal polynomials with special properties, approximation by lacunary Bernstein polynomials, weighted polynomials, homogeneous polynomials and rational function approximation. In the last two decades his main interests are on the study of polynomial inequalities on multivariate domains, convex bodies and star-like domains. Another interesting topic on which he worked, is the construction of "needle" polynomial spaces, with applications, in particular to cubature and norming-sets.

It is worth mentioning the paper A. Kroó, *On optimal polynomial meshes*, J. Approx. Th. 163 (2011), 1107-1124, in which he studied a constructive way of generating optimal meshes with respect the sup-norm on various domains in \mathbb{R}^d .



Figure 1: András during a colloquium in 2013

2 Relationship with the people of the CAA-group

As mentioned above, András has worked on polynomial inequalities, a topic of interest to a number of the members of the Constructive Approximation and Applications (CAA) research group (between the Universities of Padova and Verona). The group has a rich production of papers that concerns constructive techniques for generating meshes in various multivariate domains by using the sup and the p -norm (see the web page <https://www.math.unipd.it/~marcov/CAA.html>). For this reason András was an invited speaker at the Dolomites Workshops on Constructive Approximations in 2012 where he gave a very

^aGuest editors of the special issue: Department of Mathematics "Tullio Levi-Civita", University of Padova (Italy)

interesting talk: "Density of Multivariate Polynomials on Convex and Star like domains". He is also one of the editors of *Dolomites Research Notes on Approximation* (DRNA), the journal that since 2008 is managed by the CAA-group.

In 2017, he was visiting professor for three months at the Department of Mathematics "Tullio Levi-Civita" and subsequently again in 2019 for a month. The visits were very fruitful, both from the scientific and friendship points of view. In fact, he likes to come to Padova for research purposes and not only. He also enjoyed spending several pleasant evenings with good *red* wines produced in the region and Padova area.

For all these reasons, we thought it was a good opportunity and a nice way to celebrate Andrà's 65th birthday, by dedicating this special issue of Volume 12 (2019) of the DRNA.

Andràs, we wish you all the best for your future life, that we are sure will be fruitful, bright and full of joy.

Once again *Happy birthday!*

Padova, October 25, 2019

Stefano De Marchi and Marco Vianello